Discrete and Computational Geometry, SS 14
Exercise Sheet "6": Abstract Voronoi Diagrams II
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 27th of May, 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 16: Randomized Incremental Algorithm for Abstract Voronoi Diagrams (History Graph) (4 Points)

Consider an admissible bisecting curve system $(S, \mathcal{J})$, and make a general position assumption that no four curves in $\mathcal{J}$ intersect at the same point. Let $s_1, s_2, \ldots, s_n$ be a random sequence of S, and let R^i be $\{\infty, s_1, s_2, \ldots, s_i\}$. Please develop a randomized algorithm to construct the abstract Voronoi diagram $V(S)$ by computing $V(R^2), V(R^3), \ldots, V(R^n)$ iteratively using the history graph. In other words, for $i \geq 2$, obtain $V(R^{i+1})$ from $V(R^i)$ by insertion s_{i+1}. Let a configuration be a Voronoi edge of $V(R^i)$, for $2 \leq i \leq n$

1. Define the parent and child relation between a configuration in $V(R^i) \setminus V(R^{i+1})$ and a configuration in $V(R^{i+1}) \setminus V(R^i)$
2. Please prove that if a site conflicts a configuration, there exists a path from the root of the history graph to the configuration along which all configuration is in conflict with the site.
3. Prove that the expected time complexity of inserting s^i is $O(\log i)$

Exercise 17: Removal of General Position Assumption (4 Points)

Consider an admissible bisecting curve system $(S, \mathcal{J})$ without the general position assumption that no four curves in $\mathcal{J}$ intersect at the same point. In other words, more than three curves in $\mathcal{J}$ can intersect at the same point, and the degree of a Voronoi vertex can be more than three. Please complete the following

- Use a constant number of sites to define a Voronoi edge, i.e., formulate a configuration for a Voronoi edge. Note that a site can appear more than once in a configuration.
- Please describe how to update the conflict graph after inserting s into $V(R)$.

Exercise 18: Karlsruhe metric (4 Points)

The Karlsruhe metric, also known as the Moscow metric, is a distance measure in a radial city where there is a city center, and roads either circumvent the center or are extended from the center. The distance $d_K(p_1, p_2)$ between two points is $\min(r_1, r_2) \times \delta(p_1, p_2) + |r_1 - r_2|$ if $0 \leq \delta(p_1, p_2) \leq 2$ and $r_1 + r_2$, otherwise, where (r_i, ψ_i) are the polar coordinates of p_i with respect to the center, and $\delta(p_1, p_2) = \min(|\psi_1 - \psi_2|, 2\pi - |\psi_1 - \psi_2|)$ is the angular distance between the two points. Please prove the bisecting curve system in the Karlsruhe metric to be admissible.