Discrete and Computational Geometry, SS 14
Exercise Sheet "8": Brunn-Minkowski Inequality
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday June 17th, 14:00 pm**. *There will be a letterbox in the LBH building, close to Room E01.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 22: Inequalities and Convexity (4 Points)

1. Show that from $xp^4 \geq mp^2 - 3np$ and $p = \frac{4n}{m}$ we conlude $x \geq \frac{1}{64}\frac{m^3}{n^2}$ (Proof of Theorem 15).
2. Show that from $\frac{1}{64}\frac{|E|^3}{m^2} - m \leq \mathrm{cr}(G) \leq \binom{n}{2}$ we conclude $|E| \in O\left(n^{\frac{2}{3}}m^{\frac{2}{3}} + m\right)$ (Proof of Theoerm 13).
3. Show that the set $C' = \bigcup_{t\in[0,1]}\{t\} \times \{(1-t)A \oplus tB\}$ for convex sets A and B is also convex (Proof of Lemma 34).

Exercise 23: Concavity of volume functions (4 Points)

Let $\oplus$ denote the Minkowski sum operator.

Let $A \subset \mathbb{R}^d$ denote a single point and $B \subset \mathbb{R}^d$ denote a unit hypercube.

a) Give the formula of the volume function $v(t) = \mathrm{vol}\left((1-t)A \oplus tB\right)$.

b) Show that $v(t)^\beta$ is *not* concave on $[0,1]$ for $\beta > \frac{1}{d}$.

Exercise 24: Volume of Minkowski sums (4 Points)

1. Give a short proof of the 1-dimensional Brunn-Minkowski inequality:

$$\mathrm{vol}(A \oplus B) \geq \mathrm{vol}(A) + \mathrm{vol}(B)$$

for nonempty measurable $A, B \subset \mathbb{R}$.

2. Prove or disprove: nonempty measurable sets $A, B \subset \mathbb{R}$ satisfy:

$$\mathrm{vol}(A) + \mathrm{vol}(B) \geq \mathrm{vol}(A \oplus B).$$