Randomized Algorithms for Geometric Structure

1. Analyze the expected time complexity of the randomized Quick-Sort and explain the relation between the sorting and geometric partition.

2. Define the conflict list for an interval and describe the randomized incremental version of Quick-Sort.

3. Analyze the expected time complexity of the randomized incremental version of Quick-Sort (Backward Analysis)

4. Use a randomized binary tree to develop an on-line version of quick-sort and analyze the expected time complexity

5. Prove that given a set N of n line segments with total k intersections and an i-element subset N^i of N, the expected number of trapezoids in the vertical trapezoidal decomposition $H(N^i)$ of N^i is $O(i + ki^2/n^2)$.

6. Define conflict relations between a newly inserted segment and the current trapezoidal decomposition $H(N^i)$, and describe how to insert a new segment.

7. Analyze the expected time of inserting a line segment into $H(N^i)$, and the total expected time for constructing the vertical trapezoidal decomposition.

8. Describe how to use a history graph to develop an on-line algorithm for the vertical trapezoidal decomposition and analyze the expected time complexity.

9. Please compare conflict graphs and history graphs.

10. Regarding the paper "Kenneth L. Clarkson, Kurt Mehlhorn, and Raimund Seidel Four, Results on Randomized Incremental Construction," define a configuration, conflict relations, and history, and give one example, e.g., vertical trapezoidal decomposition.

11. Please analyze the expected number of conflict relation between the configurations in H_r (the history for the first r element) and x_r (the r^{th} element

12. Please analyze the size of conflict history.

Abstract Voronoi diagram

1. Define abstract Voronoi diagrams, describe the motivation, and list several examples.

2. Let $(S, \mathcal{J})$ be a bisecting curve system. Please prove that the following assertions are equivalent.

   - If p, q, and r are pairwise different sites in S, then $D(p,q) \cap D(q,r) \subseteq D(p,r)$ (Transitivity)
   - For each nonempty subset $S' \subseteq S$, $R^2 = \bigcup_{p \in s'} \overline{\mathrm{VR}(p, S')}$

3. Please argue that for checking an admissible system, it is enough to check all subset of 3 sites.

4. Define a conflict graph for the incremental construction of AVD, and prove that local test is enough, i.e., $e \cap \mathrm{VR}(t, R \cup \{t\}) = e \cap \mathrm{R}(t, \{p, q, r\})$, where $R \subseteq S$, $t \in S \setminus R$, and e is the Voronoi edge between $\mathrm{VR}(p, R)$ and $\mathrm{VR}(q, R)$.

5. Describe how to compute $V(R \cup \{s\})$ from $V(R)$

6. Describe how to update the conflict graph, i.e., computing $G(R \cup \{s\})$ from $G(R)$.

7. Prove that the time to compute $V(R \cup \{s\})$ and $G(R \cup \{s\})$ from $V(R)$ and $G(R)$ is linear to the number of structural changes.

8. Use the general ideas of randomized geometric algorithms to analyze the expected time complexity for the randomized incremental construction of AVD.