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Probabilistic Analysis
of Algorithms
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Problem Set 3

Problem 1

Let an instance of the knapsack problem with capacity W, weights $w_1, \ldots, w_n$, and profits $p_1, \ldots, p_n$ from the interval $[0, 1]$ be given. Recall the definition of the winner gap:

$$\Delta := p^\mathsf{T} x^* - p^\mathsf{T} x^{**},$$

where

$$x^* := \arg\max\{p^\mathsf{T} x \mid x \in \{0,1\}^n \text{ and } w^\mathsf{T} x \leq W\} \quad \text{and}$$

$$x^{**} := \arg\max\{p^\mathsf{T} x \mid x \in \{0,1\}^n \text{ and } w^\mathsf{T} x \leq W \text{ and } x \neq x^*\}.$$

We assume that there are at least two feasible solutions such that Δ is well-defined. Assume that you are given the information that $\Delta > n2^{-\ell}$ for some $\ell \in \mathbb{N}$. In what respect does this knowledge help you to find an optimal solution quickly?

Hint: Think about how rounding the profits influences the optimal solution of the problem. Remember that it is possible to solve the integer version of the knapsack problem in time $O(nP)$, where P is the sum of all profits.

Problem 2

Let I be an instance of the knapsack problem as usual, but now you are allowed to pack each item up to k times. How can this problem be reduced to an instance of the classical knapsack problem with $O(n \cdot \log k)$ many items?

Problem 3

Consider a modification of the knapsack problem where there are multiple profits for each item. This means that instead of one vector $p \in \mathbb{R}^n$ describing the profits there are multiple vectors $p^1 \ldots p^k$ and you want to maximize all $(p^i)^\mathsf{T} x$. Is it possible to generalize the Nemhauser-Ullmann algorithm to find the set of Pareto-optimal solutions for this modified problem?