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Probabilistic Analysis
of Algorithms
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Problem Set 9

Problem 1

Let $k \in \mathbb{N}$ with $k \geq 2$. Similar to the 2-Opt algorithm from the lecture, the k-Opt algorithm is a local search heuristic for the TSP problem. It also starts with an arbitrary initial tour, but instead of selecting only two edges of the current tour, k-Opt selects at most k edges of the current tour and replaces them by k other edges such that the result will be a shorter tour. We use the term k-change to denote a local improvement made by k-Opt. The algorithm terminates in a local optimum in which no further improving step is possible.

(a) Prove a result similar to Lemma 6.5 for the k-Opt algorithm.

(b) Use part (a) to prove a result similar to Theorem 6.4 for the k-Opt algorithm.

(c) Similar to Theorem 6.6, Lemma 6.7, and Lemma 6.8 use linked pairs of k-changes to improve the results from part (b).

Problem 2

Show that every TSP-instance V with an arbitrary distance function $\text{dist} : V \times V \to \mathbb{R}_{\geq 0}$ can be transformed into a TSP-instance on the same set V of vertices with a metric distance function $\text{dist}_{\text{metric}} : V \times V \to \mathbb{R}_{\geq 0}$ such that the 2-Opt state graphs are identical for both distance functions.