Selected Topics in Algorithmics, SS15
Exercise Sheet "3": Voronoi Diagrams and Delaunay Triangulations
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Wednesday 3rd of June, 14:30 pm**. *There is a letterbox in front of room E.01 in the LBH building.*
- *You may work in groups of at most two participants.*

Exercise 7: Local Test (Voronoi Diagrams) (4Points)

Consider a set S of n points. Let R be a subset of S, and let e be the Voronoi edge between $\mathrm{VR}(p, R)$ and $\mathrm{VR}(q, R)$. Please prove for a site $t \in S \setminus R$, $e \cap \mathrm{VR}(t, R \cup \{t\}) = e \cap \mathrm{VR}(t, \{p, q, t\})$. (Hint: prove $e \cap \mathrm{VR}(t, R \cup \{t\}) \subseteq e \cap \mathrm{VR}(t, \{p, q, t\})$ and $e \cap \mathrm{VR}(t, \{p, q, t\}) \subseteq e \cap \mathrm{VR}(t, R \cup \{t\})$. The former is obvious, so you can consider any point in $e \cap \mathrm{VR}(t, \{p, q, t\})$ to prove the latter. A point belongs to $\mathrm{VR}(t, R \cup \{t\})$ if its closest site in $R \cup \{t\}$ is t. Any point in e is closer to p and q than other points in R.)

Exercise 8: Voronoi Diagrams by History Graph (6Points)

Consider a set S of n points satisfying the general position assumption that no three points are in the same line, and no four points are in the same circle. Let $s_1, s_2, \ldots, s_n$ be a random sequence of S, and let R^i be $\{s_1, s_2, \ldots, s_i\}$. Please develop a randomized algorithm to construct the Voronoi diagram $V(S)$ of S by computing $V(R^5), V(R^6), \ldots, V(R^n)$ iteratively using the history graph. In other words, for $i \geq 4$, obtain $V(R^{i+1})$ from $V(R^i)$ by adding s_{i+1}. (Hint: Transform the randomized incremental construction using conflict lists in the slide into one using history graph.)

1. Define a node in the history graph.

2. Describe the parent and child relation in the history graph. (3 points) (Hint: If a point conflicts with a child, the point should conflict with one parent of the child.)

3. Describe the insertion of s_{i+1} using the history graph.

4. Prove the expected cost of inserting s^{i+1} to be $O(\log i)$ and the expected cost of construction $V(S)$ to be $O(n \log n)$

Exercise 9: Delaunay Triangulation by Conflict Lists (6Points)

Consider a set S of n points satisfying the general position assumption that no three points are in the same line, and no four points are in the same circle. Let $s_1, s_2, \ldots, s_n$ be a random sequence of S, and let R^i be $\{s_1, s_2, \ldots, s_i\}$. Please develop a randomized algorithm to construct the Delaunay triangulation $\mathrm{DT}(S)$ of S by computing $\mathrm{DT}(R^5), \mathrm{DT}(R^6), \ldots, \mathrm{DT}(R^n)$ iteratively using the conflict lists. In other words, for $i \geq 4$, obtain $\mathrm{DT}(R^{i+1})$ from $\mathrm{DT}(R^i)$ by adding s_{i+1}. (Hint: Use the duality between the Voronoi diagram and the Delaunay triangulation.)

1. Define a configuration which you will use.

2. Define a conflict relation between a configuration and a point.

3. Describe the insertion of s_{i+1} using the conflict lists. (3 points)

4. Prove the expected construction time to be $O(n \log n)$