<div style="border:1px solid">

Selected Topics in Algorithmics, SS15
Exercise Sheet "4": Configuration Space and Random Sampling
University of Bonn, Department of Computer Science I

</div>

- *Written solutions have to be prepared until* **Wednesday 24th of June, 14:30 pm**. *There is a letterbox in front of room E.01 in the LBH building.*

- *You may work in groups of at most two participants.*

Exercise 10:   **3D Convex Hulls**   **(6 Points)**

Given a set N of n half-spaces in three dimensions, a convex hull $G(N)$ of N is the intersection of half-spaces in N. Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. We assume a general position assumption that for every three half-spaces in N, their defining hyperplanes intersect at exactly one point, and for every four half-spaces in N, the intersection of their defining hyperplanes is empty.

- What is a feasible configuration for $G(R)$, where R is a subset of N?

- What is the maximum degree of such a configuration and what is the expected number of configurations for $G(R)$? (Hint: the number of vertices, edges, and faces in $G(R)$ is $O(r)$.)

- What is the expected structural change during the incremental construction?

- What is the expected conflict change during the incremental construction?

Exercise 11:   Random Sampling for Trapezoidal Decomposition (4 Points)

Given a set N of n line segments in the plane, let $H(N)$ be the vertical trapezoidal decomposition formed by line segments in N. We make a general position assumption that the x-coordinate of an endpoint or the intersection between two line segments is unique. Consider a random sample R of N of size r. Please prove that with high probability, for each trapezoid $\triangle$ in $H(R)$, the number of line segments in $N \setminus R$ that intersect $\triangle$ is bounded by

$$O(\frac{n}{r-4} \log r).$$

Bonus 1: Update Conflict List for Delaunay Triangulation (5 points)

In Exercise 9, we describe the conflict relation between a configuration and a point. Here, please describe how to update conflict lists due to the insertion of a point in time proportional to the total size of conflicts of newly created and destroyed configurations. Recall that a configuration is a Delaunay edge together with its two adjacent triangles.