

Online Motion Planning, SS 16  
Exercise sheet 6  
University of Bonn, Inst. for Computer Science, Dpt. I

- *You can hand in your written solutions until Wednesday, 01.06., 14:15, postbox in front of room E.01 LBH.*

**Exercise 16:      Comparison of Bug-variants                      (4 points)**

- a) Present an example where strategy Bug1 beats the strategy Bug2 w.r.t. path length.
- b) Show the tightness of the three presented Bug variants Bug1, Bug2 and ChangeI, i.e., show by examples that you can get arbitrarily close to the presented path length.

**Exercise 17:      Variant of 2-ray search                                      (4 points)**

We consider the following variant of the 2-ray search for a target point. The corresponding unknown target point  $t$  is located on two rays which build a right angle at the common source  $s$  as shown in Figure 1.

The agent starts at  $s$  and detects the unknown target  $t$  only by touching it. For moving back from one ray to the other the agent can move in the *free space*. Figure 1 shows such a short-cut. Note that a reasonable strategy has to visit the point on both rays consecutively by increasing distance.

- a) Describe a reasonable strategy and its local worst-case situation by functionals in analogy to the standard 2-ray case.
- b) Find the optimal strategy by application of the Theorem of Gal. Just assume that the conditions of the Theorem hold.

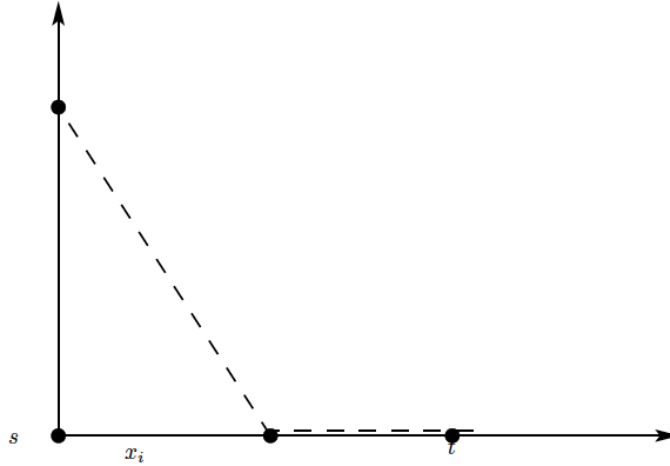


Figure 1: In this variant, for the path back, the agent can move in the free-space.

**Exercise 18: Proof the Gal-Theorem conditions (4 points)**

For  $\theta \in (0, \pi/4]$  we consider the functionals

$$F_k(x_1, x_2, \dots, x_{k+1}) := \frac{\sum_{i=1}^k \sqrt{x_i^2 + x_{i+1}^2 - 2 \cos(\theta) x_i x_{i+1}}}{x_k}.$$

- a) Proof that unimodality holds, i.e.:

$$F_k(A \cdot X) = F_k(X) \text{ and } F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$$

holds for  $A > 0$  and sequences  $X$  and  $Y$ .

- b) Make use of the Theorem of Gal and define the function  $f(a)$  that has to be optimized for  $a > 1$ . Try to find a simple representation of  $f$ .