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3.4 Optimal search paths

We now consider the problem of searching for a goal in more general environments such as polygons (with and without holes) graphs or trees. We consider the online and offline version. In the offline version the environment is known, the goal remains unknown. In any case a search path has to visit or *see* all possible goals of the given environment. Therefore any search path is also an exploration path for all goals.

If the agent has a vision system, for any goal t there will be a first point on the search path where t gets visible. After reaching this point the agent can move to t along the shortest path. The agent will only use this last path, if t is the goal. In this sense the search path itself does not visit the goals, it only *sees* any goal. On the contrary, if the agent has no vision, the search path has to visit any possible goal.

In the online version of the problem, additionally the agent has to gain more information about the enviroment for future computation. We have already seen that a general constant competitive search strategy does not exists for all groups of environments. For example, searching for the goal among m fixed corridors with a cave at distance 1 at the end of each corridor, results in a search path of length $2m-1$ whereas the shortest path to the last seen point is 1. Since m can go to infinity, there is no constant C for the ratio. On the other hand for the fixed configuration of m corridors (m is fixed) no other strategy than visiting all caves successively has a better ratio. This means that there is a large ratio for the last point visited against the shortest path, but any strategy has this large ratio. Therefore for a comparisons between good or bad search paths we just compare a path to the worst case ratio that any strategy has to cope with.

The following definition is made for arbitrary environments $\mathcal{E}$. For trees and graphs $G=(V,E)$ we consider two different variants w.r.t. the goal set. In the **vertex search** variant, the goals can only be located at the vertices of G. The agent need not necessarily visit all edges. In the **geometric search** variant, the goal can be located everywhere on the graph. Any search path has to visit all edges and vertices.

For the general definition we introduce the **goal set** $\mathcal{G}\subseteq\mathcal{E}$. For a graph $G=(V,E)$ and the vertex search problem we have $\mathcal{G}=V$, and $\mathcal{G}=V\cup E$ for the geometric search.

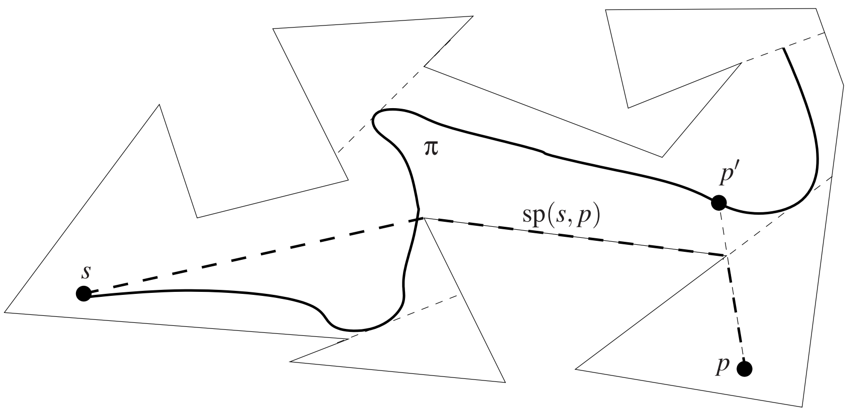

Figure 3.27: A search path π in a simple polygon. The point p' on π, is the first point on π such that p is *seen* from π.

Definition 3.19 Let $\mathcal{E}$ be an environment, $\mathcal{G}\subseteq\mathcal{E}$ a **goal set** and s a point inside $\mathcal{E}$. A **search path** π with start point s is a path in $\mathcal{E}$, such that π starts in s and detects any point of $\mathcal{G}$ at least once. The **performance** of the search path (denoted as search ratio) is defined by[4]

$$\mathrm{SR}(\pi) := \max_{p\in\mathcal{G}} \frac{|\pi_s^{p'}|+|p'p|}{|\mathrm{sp}(s,p)|},$$

[4] ${\pi_{\mathrm{Strat}}}_a^b$ denotes the portion of π_{Strat} from a to b, $\mathrm{sp}(a,b)$ denotes the shortest path from a to b in $\mathcal{E}$.

where p' denotes the first point along π, such that p is detected from π; compare Figure 3.27. An **optimal search path** π_{opt} is a search path for E and $\mathcal{G}$ with minimal search ratio over all search paths in $\mathcal{E}$.

For agent without visiom we have $p' = p$ for all points of $\mathcal{G}$ from $\mathcal{E}$. The performance is given by:

$$\mathrm{SR}(\pi) = \max_{p \in \mathcal{G}} \frac{|\pi_s^p|}{|\mathrm{sp}(s,p)|}.$$

For some environments, it is known that the computation of the optimal search path is an NP-hart problem, i.e. for graphs [KPY96]. For some other environments it is even not clear how the optimal search path can be computed. Computing an optimal search path is a difficult task, therefore we are looking for good and easy to compute approximations. We would also like to approximate the optimal search path in the online version, i.e., we are looking for a constant C_S, such that for any environment we guarantee

$$\mathrm{SR}(\pi_{\text{onl}}) \leq C_S \cdot \mathrm{SR}(\pi_{\text{opt}}).$$

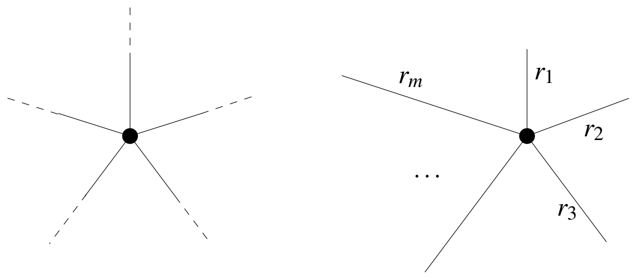

Figure 3.28: (i) m ray, (ii) m segments of different length.

At first place the optimal search path is computed for the offline version and can be handled as a comparison measure for the online version. If the offline optimal search path is not known, any online approximation is also an offline approximation. Let us consider some examples:

1. If we are searching for a goal on m rays, that emanate from a common start point s, the online and the offline version coincidence. We do not have more information in the offline version.. The search strategy that visits ray $(i \bmod m)$ with depth $\left(\frac{m}{m-1}\right)^i$ in the i-the step has the best competitve ratio among all possible strategies. Therefore trivially this is also the optimal search path. This means we have an approximation of the search ratio by factor $C_S = 1$.

2. If we slightly relax the above example and replace the m rays by segments of different length $r_1, r_2, \ldots, r_m$ as show in Figure 3.28, the problem of computing the optimal search path is still unknown for the offline case. We have to compute the optimal competitive strategy in the offline version. Approximations are possible by applying and adjusting the optimal competitive strategy of the ray version (all $r_i = \infty$).

3. In case of a tree environment the optimal search path for the vertex search can be computed in exponential time, when the tree is given. We consider any permutation of the vertex set and calculate the search ratio of the corresponding path. So we will find the best search path.

4. For the geometric search in trees we cannot apply the above (example 3) strategy since the goal might be located along an edge. Example 2 is a special case of the geometric search version on general trees, therefore the optimal offline search path is still unknown also for general trees in this case.

Algorithm 3.2 Searchpath by doubling exploration depth

- Let $\mathrm{Expl}_{\mathrm{onl}}(d)$ be a competitive (online) strategy for the exploration of an environment up to depth d. The strategy finally returns to the start.
- Successivly explore environment $\mathcal{E}$ by increasing depth, applying $\mathrm{Expl}_{\mathrm{onl}}(2^i)$ for $i = 1, 2, \ldots$ from start point s.

Some of the above example cry for an approximation of the search ratio. The same holds for the online version. The general idea for an approximation is as follows. For the given environment we successively apply a constant competitive (online or offline) exploration strategies with increasing depth $d = 2^i$ in analogy to the doubling heuristic used by searching for the door along a wall (doubling heuristic; see Algorithm 3.2). Let $\mathrm{Expl}_{\mathrm{onl}}$ denote a competitive online strategy that explores the environment and returns to the start. Let $\mathrm{Expl}_{\mathrm{onl}}(d)$ denote a sub-strategy that performs an exploration restricted to all goals in the (possibly larger) environment that are no more than distance d away from the start. $\mathrm{Expl}_{\mathrm{opt}}$ and $\mathrm{Expl}_{\mathrm{opt}}(d)$ denote the corresponding offline strategies for these problems. Furthermore, $\pi_{\mathrm{Expl}_{\mathrm{onl}}}$ etc. denote the corresponding paths.

Lemma 3.20 *Let $\mathcal{E}$ be an environment, such that an agent without vision system is searching for a goal. Let $\mathrm{Expl}_{\mathrm{onl}}(d)$ be a C-competitive strategy that explores $\mathcal{E}$ up to distance d. By the use of the doubling heuristic (Algorithm 3.2) we achieve a $4C$-approximation of the optimal search ratio.*

Proof. $\mathrm{Expl}_{\mathrm{onl}}(d)$ is competitive, which means that there is a constant C such that for all enviroments $\mathcal{E}$ we have

$$|\pi_{\mathrm{Expl}_{\mathrm{onl}}(d)}| \leq C \cdot |\pi_{\mathrm{Expl}_{\mathrm{opt}}(d)}| . \tag{3.14}$$

Also the optimal search path π_{opt} finally visits all points with distance d. Let $\mathrm{last}(d)$ be the last point at distance d from s that is detected (and visited) by π_{opt}. The performance of this point is $\frac{|{\pi_{\mathrm{opt}}}_s^{\mathrm{last}(d)}|}{d}$. This is a lower bound of the search ratio, the general performance of π_{opt} cannot be better than the performance ratio in $\mathrm{last}(d)$. Therefore we give the following lower bound:

$$\mathrm{SR}(\pi_{\mathrm{opt}}) \geq \frac{|{\pi_{\mathrm{opt}}}_s^{\mathrm{last}(d)}|}{d}. \tag{3.15}$$

The optimal search path π_{opt} applied from s to $\mathrm{last}(d)$ explores all goals at depth d and ${\pi_{\mathrm{opt}}}_s^{\mathrm{last}(d)}$ is an exploration path for depth d. If we return from $\mathrm{last}(d)$ to the start s by the shortest path of length d, we obtain an exploration tour that returns to the start. This overall path is not shorter that the optimal depth d restricted exploration tour (with return to s), we conclude

$$|\pi_{\mathrm{Expl}_{\mathrm{opt}}(d)}| \leq |{\pi_{\mathrm{opt}}}_s^{\mathrm{last}(d)}| + d. \tag{3.16}$$

From Equation 3.15 and Equation 3.16 we have

$$|\pi_{\mathrm{Expl}_{\mathrm{opt}}(d)}| \leq d \cdot (\mathrm{SR}(\pi_{\mathrm{opt}}) + 1). \tag{3.17}$$

The strategy applies $\mathrm{Expl}_{\mathrm{onl}}(d)$ with increasing exploration depth $d = 2^0, 2^1, 2^2, \ldots$ The worst case for a ratio is attained, if we muss a goal with distance $2^j + \varepsilon$ in the round for exploration depth $d = 2^j$

and detect and visit this point almost at the end of the round with exploration depth $d = 2^{j+1}$. This gives a worst case search ratio for each round by

$$\begin{aligned}
\mathrm{SR}(\pi) &\leq \frac{\sum_{i=1}^{j+1} |\pi_{\mathrm{Expl}_{\mathrm{onl}}(2^i)}|}{2^j + \varepsilon} \\
&\underset{(3.14)}{\leq} \frac{C}{2^j} \sum_{i=1}^{j+1} |\pi_{\mathrm{Expl}_{\mathrm{opt}}(2^i)}| \\
&\underset{(3.17)}{\leq} \frac{C}{2^j} \sum_{i=1}^{j+1} 2^i \cdot (\mathrm{SR}(\pi_{\mathrm{opt}}) + 1) \\
&\leq C \cdot \left(\frac{2^{j+2} - 1}{2^j} \right) \cdot (\mathrm{SR}(\pi_{\mathrm{opt}}) + 1) \leq 4C \cdot (\mathrm{SR}(\pi_{\mathrm{opt}}) + 1).
\end{aligned}$$

□

For trees we have an optimal exploration strategy for any d with a ratio of $C = 1$ by DFS:

Corollary 3.21 *(Koutsoupias, Papadimitriou, Yannakakis, 1996)*
For any tree we can approximate the optimal search ratio by a factor of 4.
[KPY96]

An interesting result, because the optimal search path and the optimal search ratio is unknown.

For other environments, the main problem is finding competitive strategies for the depth restricted exploration. For general graphs $G = (V, E)$ we have introduced CFS-Algorithm in Section 1.5 on page 31. This algorithm can be used for depth restricted exploration for depth $r := d$. There is a problem with this strategy, since we use a rope of length $(1+\alpha)d$ and guarantee a competitive factor of $4 + \frac{8}{\alpha}$, we guarantee the exploration only for depth d.

Since we explore the graph with rope length $(1+\alpha)d$ it might happen that also parts of the graph with distance larger than d will be explored. In the offline optimal exploration path such parts will never be visited. The workaround for this problems is as follows. We compare the restricted depth strategy with for depth d (that partially visits depth βd) to the optimal offline exploration with depth βd. In this case we are on the safe side. In the case of CFS we have $\beta = 1 + \alpha$. Also the comparison ratio might depend on β. We make use of a ratio C_β such that $\mathrm{Expl}_{\mathrm{onl}}(d) \leq C_\beta \cdot \mathrm{Expl}_{\mathrm{opt}}(\beta d)$ holds. For the CFS we have $C_\beta = 4 + \frac{8}{\alpha}$.

Theorem 3.22 *(Fleischer, Kamphans, Klein, Langetepe, Trippen, 2003)*
Let $\mathcal{E}$ be an environment where an agent without vision system is searching for a target. Let $\mathrm{Expl}_{\mathrm{onl}}(d)$ *be strategy for the depth restricted online exploration of $\mathcal{E}$ with* $\mathrm{Expl}_{\mathrm{onl}}(d) \leq C_\beta \cdot \mathrm{Expl}_{\mathrm{opt}}(\beta d)$. *We can search in $\mathcal{E}$ by the doubling heuristic (Algorithm 3.2) and attain a eine $4\beta C_\beta$-approximation of the optimal search path and search ratio.* *[FKK+04]*

Proof. In pure analogy to the proof of Lemma 3.20, only changing the version of Equation 3.14. □

Corollary 3.23 *For general graphs and online geometric search we can approximate the optimal search ratio by a factor of* $4(1+\alpha)(4+\frac{8}{\alpha})$.

In the above version without vision we always guaranteed that the last point, $\mathrm{last}(d)$, detected at distance d is also exactly visited at this moment in time. For an agent with a vision system it might happen that the search paths visits a point $\mathrm{last}(d)$ from which the last point at distance d is detected and seen but $\mathrm{last}(d)$ has not distance d to the start.

We can no longer conclude $|\mathrm{sp}(s, \mathrm{last}(d))| \leq d$, which was required for the bound Equation 3.15. Fortunately, for the agent with vision system we can at least guarantee that $|\mathrm{sp}(s, \mathrm{last}(d))| \leq |\pi_{\mathrm{opt}_s}^{\mathrm{last}(d)}|$ holds. For moving back to the start from $\mathrm{last}(d)$ we can use the same path back.

This gives a different lower bound for the optimal search ratio against the optimal offline exploration tour, which is:

$$\mathrm{SR}(\pi_{\mathrm{opt}}) \geq \frac{|\pi_{\mathrm{opt}_s}^{\mathrm{last}(d)}|}{d} \geq \frac{|\pi_{\mathrm{Expl}_{\mathrm{opt}}(d)}|}{2d} \Longleftrightarrow |\pi_{\mathrm{Expl}_{\mathrm{opt}}(d)}| \leq 2d \cdot \mathrm{SR}(\pi_{\mathrm{opt}}).$$

Altogether we attain a factor of

$$\begin{aligned}\frac{\sum_{i=1}^{j+1} |\pi_{\mathrm{Expl}_{\mathrm{onl}}(2^i)}|}{2^j} &\leq C_\beta \cdot \frac{\sum_{i=1}^{j+1} |\pi_{\mathrm{Expl}_{\mathrm{opt}}(\beta 2^i)}|}{2^j} \leq 2C_\beta \cdot \frac{\sum_{i=1}^{j+1} \beta 2^i \, \mathrm{SR}(\pi_{\mathrm{opt}})}{2^j} \\ &\leq 8\beta C_\beta \cdot \mathrm{SR}(\pi_{\mathrm{opt}})\,.\end{aligned}$$

Theorem 3.24 *Let $\mathcal{E}$ be an environment where an agent with vision system is searching for a target. Let* $\mathrm{Expl}_{\mathrm{onl}}(d)$ *be strategy for the depth restricted online exploration of $\mathcal{E}$ with* $\mathrm{Expl}_{\mathrm{onl}}(d) \leq C_\beta \cdot \mathrm{Expl}_{\mathrm{opt}}(\beta d)$. *We can search in $\mathcal{E}$ by the doubling heuristik (Algorithm 3.2) and attain a eine* $8\beta C_\beta$*-approximation of the optimal search path and search ratio.* *[FKK+04]*

With this general framework we can approximate optimal search path for polygons also in an online fashion. The main task is the design of exploration strategies which will be the subject of the next section.

For the negative side we will now show some examples where an agent (without a vision system) cannot approximate the offline optimal search path with a constant factor in the online version. Lower bounds are achieved by counter examples. For some graph configuration we show that the search ratio is constant (the competitive ratio is small) whereas any online strategy can be forced to make arbitrary large detours against the shortest path to some goals. In comparison Corollary 3.23 for the geometric search has used the property that the CFS Algorithm has running time of $(4+8/\alpha)|E(d)|$ for depth restricted exploration. If the goal set is restricted to the vertices, the result will not help us anymore.

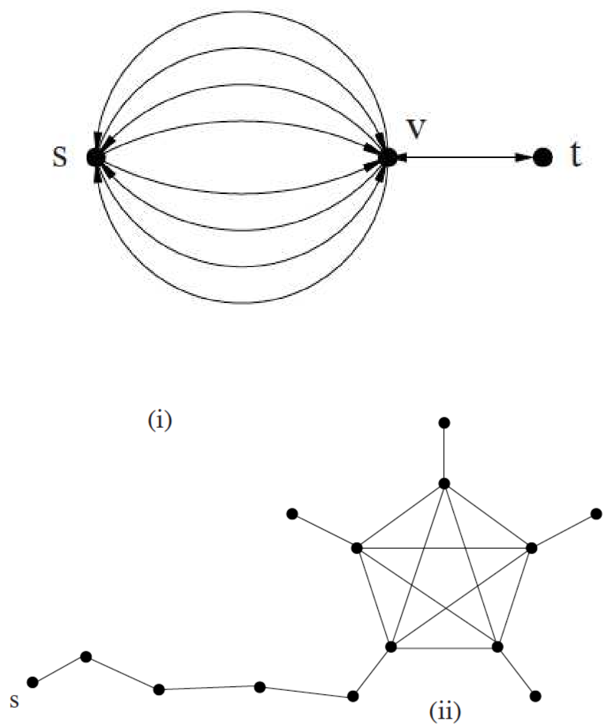

Figure 3.29: The optimal search path for goal set V cannot be approximated by a constant factor for (i) planar graphs with multiple edges and (ii) general graphs without multiple edges.

Theorem 3.25 *For the following graph configuration we can show that otimal offline search path cannot be approximated by an online search startegy with a constant factor.*

1. *Planar graphs $G = (V, E)$ with multiple edges and goal set V.*
2. *General graphs $G = (V, E)$ even without multiple edges and goal set V.*
3. *Directed graphs $G = (V, E)$ with goal set $E \cup V$.*

Exercise 21 *Show that for directed graphs $G = (V, E)$ with goal set $E \cup V$ a constant approximation of the optimal search path and search ratio is not possible.*

Exercise 22 *Consider planar graphs $G = (V, E)$ with goal set V. Does a constant approximation of the optimal search path and search ratio exist?*

Proof.

1. In Figure 3.29(i) the optimal search path visits vertices v and t with search ratio 1. Any online strategy will be forced to visit all multiple edges before t is visited. This gives a ratio of $\frac{k}{2}$ for arbitrary k.
2. In Figure 3.29(ii) the optimal search path visit the satellites of the k-clique from s in $3k$ steps. The distance from s to the clique is also k. This gives a search ratio of at most 4. An online strategy will be forced to visit all inner edges first, before the satellites will be visited. Therfore $\Omega(k^2)$ steps will be required and the search ratio is $\Omega(k)$.

□

The next paragraph will handle exploration strategies by an agent with a vision system. An analogous negative result (no constant competitive search path approximation) will be achieved for polygons with obstacles (or holes).

Interestingly, for all negative examples, there is already no constant competitive online exploration strategy for the corresponding goal set. This is extended to the negative result for the search path approximation. Altogether, the conjecture is that both statements are equivalent in general

Already proved: $\exists$ constant-competitive, (depth restricted) exploration strategy $\Rightarrow \exists$ online search strategy with constant search ratio approximation.

Conjecture: $\nexists$ constant-competitive exploration strategy and $\exists$ 'extentable' lower bound $\Rightarrow \nexists$ online search strategy competitive against search ratio.

An extension trick for the lower bound can be seen in Figure 3.29(ii), the path to the k-clique was extended such that the search ratio of the optimal search path is constant. A similar idea is applied for polygons with holes; see Figure **??**.

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