Rheinische Friedrich-Wilhelms-Universität Bonn
Institut für Informatik I

Elmar Langetepe

Online Motion Planning

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Proof. For the proof of (i),(ii),(iii) and (v) we apply the same arguments as in the proof of Lemma 1.23. It remains to show that (iv) holds. The main difference is that the size of a tree T is directly correlated to the distance from s to T, this is different from the previous argumentation.

Let us first show that the remaining tree T_i (after pruning) will be fully explored by DFS. For any vertex v in T_i we have $d_{T_i}(s_i,v) \leq \frac{9d_{G^*}(s,s_i)\alpha}{16}$, otherwise v has been cut of by pruning. Thus we have

$$(1+\alpha)d_{G^*}(s,s_i) - d_{G^*}(s,s_i) - d_{T_i}(s_i,v) \geq \frac{7d_{G^*}(s,s_i)\alpha}{16},$$

which shows that the tether is long enough T_i will be fully explored by DFS.

By induction over the number of pruning steps we will finally show: $\forall T \in \mathcal{T} : |T| \geq \frac{\max(d_{G^*}(s,T),c)\alpha}{4}$.

In the beginning we apply bDFS from the start with tether length c. Either we explore the whole graph or we have $|T| \geq (1+\alpha)c > \frac{\alpha c}{4}$ for the resulting spanning tree T. For simplicity we assume $d_{G^*}(s,T_i) > c$ from now on.

We would like to show that for any tree T_w, resulting from the pruning of some T_i, we have $|T_w| \geq \frac{d_{G^*}(s,T_w)\alpha}{4}$. Also the remaining tree T_i has this property.

For the remaining tree T_i (after pruning), we conclude $d_{G^*}(s,T_i) = d_{G^*}(s,s_i)$ and pruning guarantees $|T| \geq \frac{d_{G^*}(s,T)\alpha}{4}$. For a tree T_w pruned from T_i we have: $|T_w| \geq \frac{9d_{G^*}(s,s_i)\alpha}{16} - \frac{d_{G^*}(s,s_i)\alpha}{4} = 5\frac{d_{G^*}(s,s_i)\alpha}{16}$ by the pruning values. Additionally, we have $d_{G^*}(s,T_w) \leq d_{G^*}(s,s_i) + d_{G^*}(s_i,w) = (1+\frac{\alpha}{4})d_{G^*}(s,s_i)$, since the root w of T_w is exactly $\frac{\alpha d_{G^*}(s,s_i)}{4}$ steps away from s. Für $0 < \alpha < 1$ we conclude: $d_{G^*}(s,T_w) < \frac{5d_{G^*}(s,s_i)}{4}$ and together with the above inequality we have $|T_w| > \frac{d_{G^*}(s,T_w)\alpha}{4}$.

Finally, we have to analyse the emerging spanning trees T_v, which will be constructed from the bDFS steps starting during the DFS walk in T_i. Such a tree T_v *starts* at some incomplete vertex v in T_i. We have $d_{G^*}(s_i,v) \leq \frac{9\alpha d_{G^*}(s,s_i)}{16}$, otherwise v would have been pruned and could not be a leaf of the rest of T_i any more. Thus we have $d_{G^*}(s,T_v) \leq d_{G^*}(s,s_i) + d_{G^*}(s_i,v) < \frac{25d_{G^*}(s,s_i)}{16}$ or $d_{G^*}(s,s_i) > \frac{16d_{G^*}(s,T')}{25}$. If T_v is fully explored, we are done, since the tree will be deleted. Assume that T_v still has incomplete vertices. As mentioned above we have $d_T(s_i,v) \leq \frac{9\alpha d_{G^*}(s,s_i)}{16}$. Starting from v there was a remaining tether length of $\frac{7\alpha d_{G^*}(s,s_i)}{16}$ for the construction of the incomplete T_v, which gives $|T_v| \geq \frac{7\alpha d_{G^*}(s,s_i)}{16}$. Application of $d_{G^*}(s,s_i) > \frac{16d_{G^*}(s,T_v)}{25}$ gives $|T_v| > \frac{7\alpha d_{G^*}(s,T_v)}{25} > \frac{d_{G^*}(s,T_v)\alpha}{4}$. Either we have explored everything behind v or the spanning tree T_v has size $|T_v| > \frac{d_{G^*}(s,T_v)\alpha}{4}$.

We have considered any emerging $T \in \mathcal{T}$! □

Theorem 1.28 *(Duncan, Kobourov, Kumar, 2001/2006)*
Applying the CFS–Algorithm with the adjustments above results in a correct restricted graph-exploration of an unknown graph with unknown depth. The algorithm is $(4+\frac{8}{\alpha})$*–competitive.* *[DKK06, DKK01]*

Proof. We apply the same analysis as in the proof of Theorem 1.24. For the analysis of the movements from s to the roots of the trees we make use of the correlation $|T_R| > \frac{d_{G^*}(s,T_R)\alpha}{4}$. □

For the number of steps we can also refine the analysis, analogously.

Corollary 1.29 *The above CFS–Algorithm for the restricted exploration of an unknown graph with unknown depth requires* $\Theta(|E|+|V|/\alpha)$ *exploration steps, which is optimal.*

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