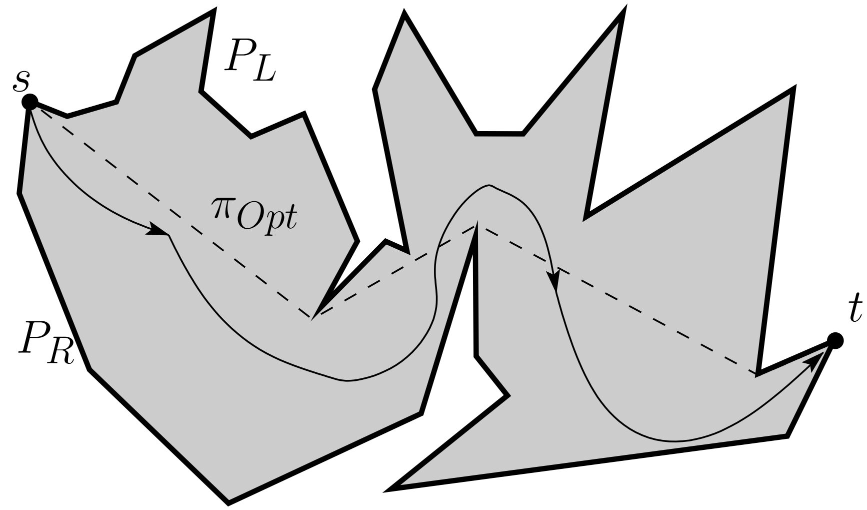
Online Motion Planning MA-INF 1314
Searching in streets!

Elmar Langetepe
University of Bonn

Rep.: Street

Def. Polygonal boundary chains P_L and P_R of P between s and t weakly visible.

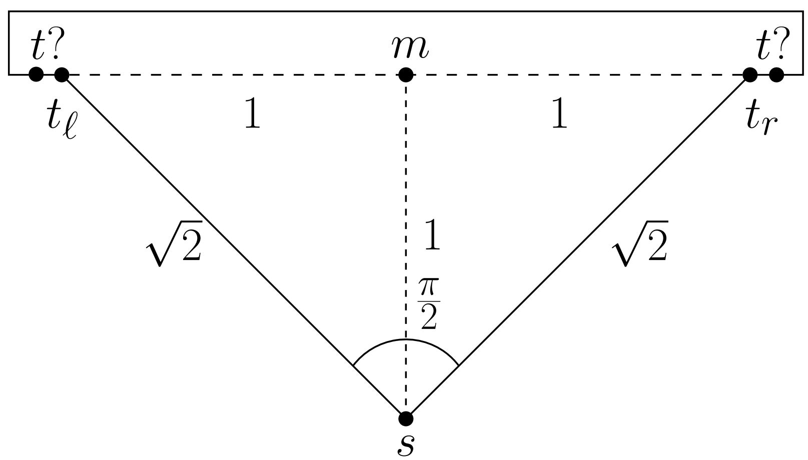
Task: Start at s, find t!



Rep.: Lower Bound!

Theorem: No strategy attains a ratio better than $\sqrt{2}$ versus the length of the shortest path.

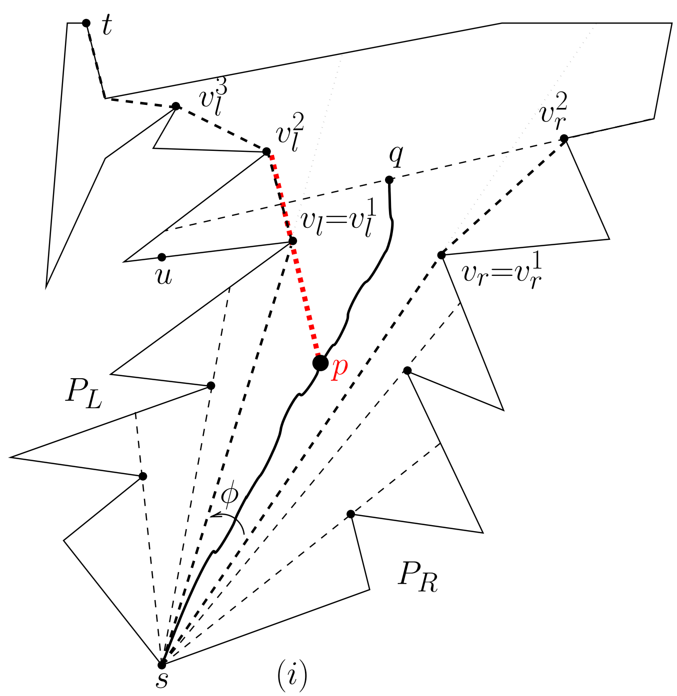
Beweis:



Detour of factor at least $\sqrt{2}$

Rep.: Reasonable strategies!

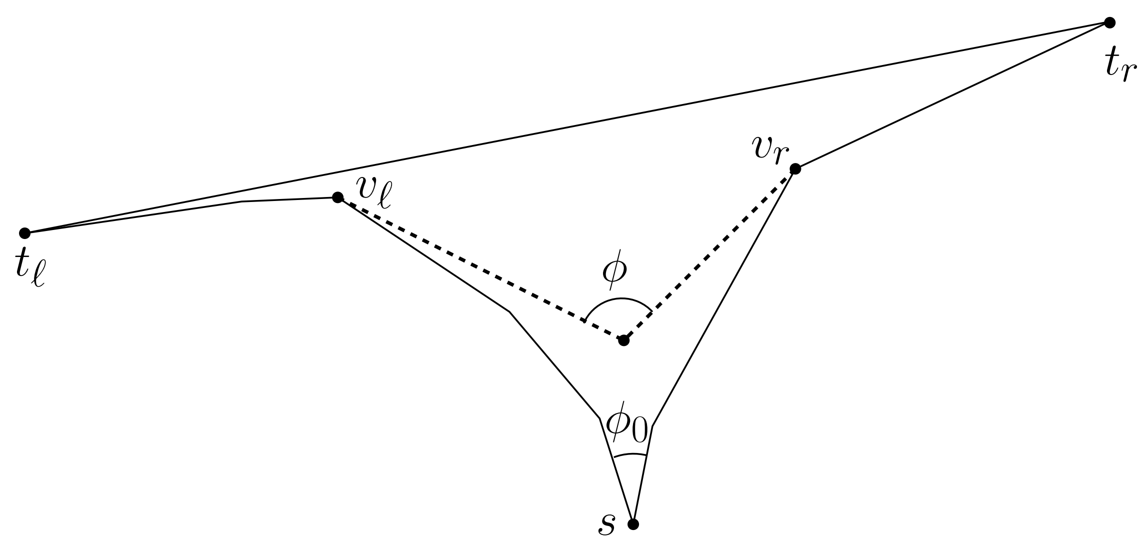
- Rightmost left reflex vertex, leftmost right reflex vertex!
- Move into the wedge of c, v_l and v_r
- One side-candidate vanishes, move directly to the other
- Extreme vertices change over time



Rep.: Funnel polygons!

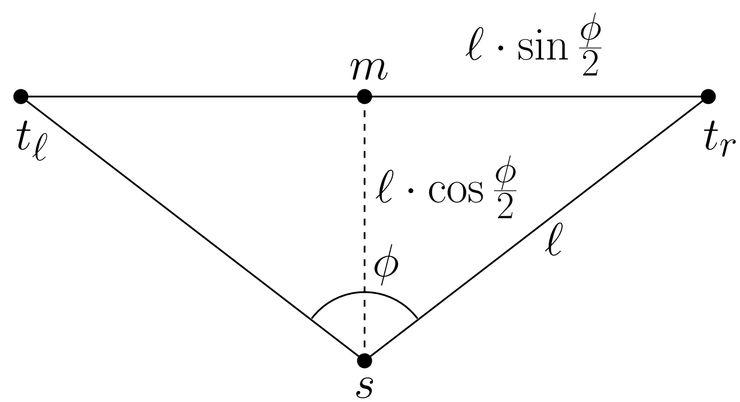
- It is sufficient to analyse special streets

- **Def.** Polygon, conve vertex s, two opening convex polygonal chains P_L and P_R starting in s ending at t_ℓ and t_r, respectively. Segment $\overline{t_\ell t_r}$ closes the funnel (polygon).



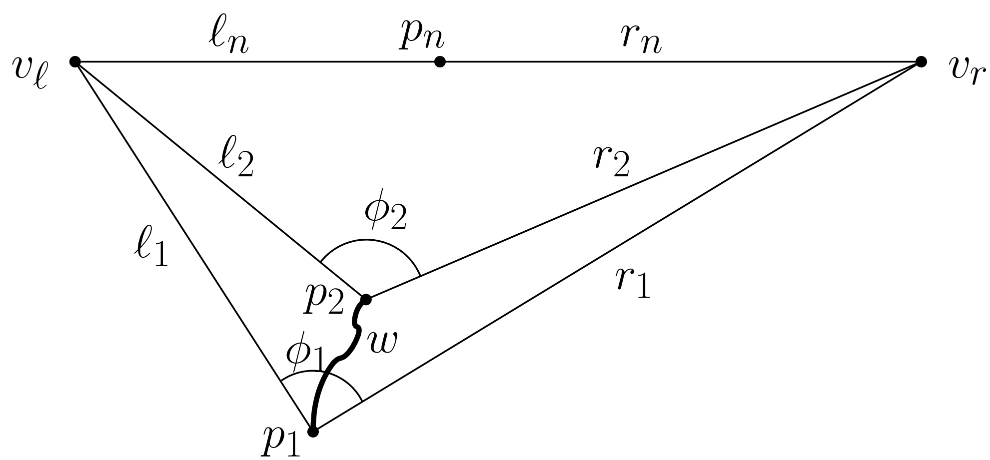
Rep.: Generalized LB for funnels

- **Lemma** LB for funnel of opening angle ϕ: $K_\phi := \sqrt{1 + \sin\phi}$.
- Strongly increasing: $0 \le \phi \le \pi/2$, Interval $[1, \sqrt{2}]$
- Strongly decreasing: $\pi/2 \le \phi \le \pi$, Interval $[\sqrt{2}, 1]$
- Subdivide: Strategy up to $\phi_0 = \pi/2$, Strategy from $\phi_0 = \pi/2$



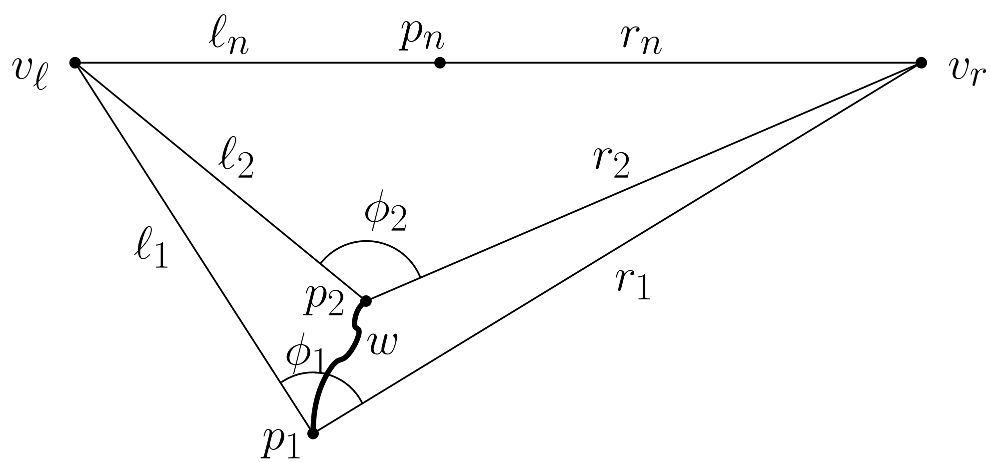
Rep.: Opt. strat. for angles $\pi \geq \varphi_0 \geq \pi/2$!

- Backward analysis: For $\varphi_n := \pi$ optimal strategy.
- $K_\pi = 1$ and K_π-competitive opt. strategy with path l_n or r_n!
- Assumption: Opt. strategy for some ϕ_2 with factor K_{ϕ_2} ex.
- How to prolong for ϕ_1 with factor K_{ϕ_1} where $\frac{\pi}{2} \leq \phi_1 < \phi_2$?
- We have $K_{\phi_1} > K_{\phi_2}$



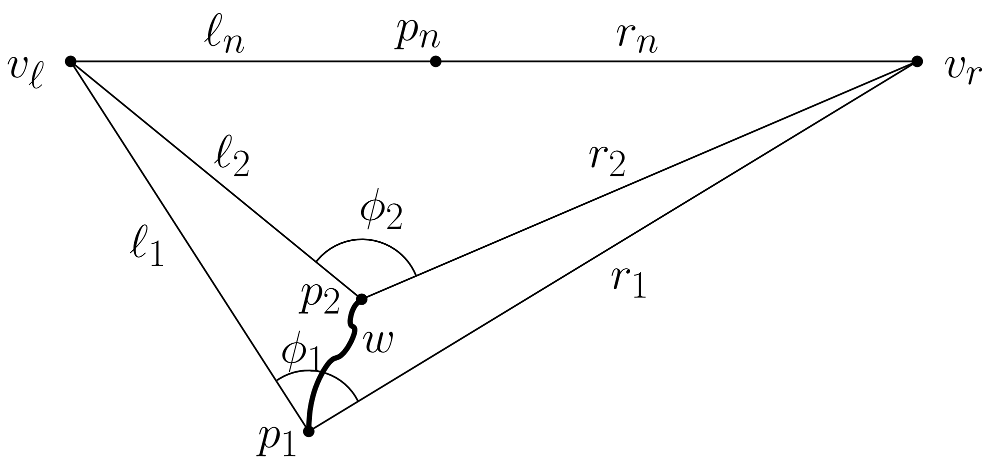
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- Situation: Opt. strategy for ϕ_2 with ratio K_{ϕ_2}
- How to get opt. strategy for K_{ϕ_1}?
- Conditions for the path w? Design!
- Goal behing v_l, path: $|w| + K_{\phi_2} \cdot \ell_2$, optimal: l_1
- Goal behind v_r, path: $|w| + K_{\phi_2} \cdot r_2$, optimal: r_1
- Means: $\frac{|w|+K_{\phi_2}\cdot\ell_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w|+K_{\phi_2}\cdot r_2}{r_1} \leq K_{\phi_1}$



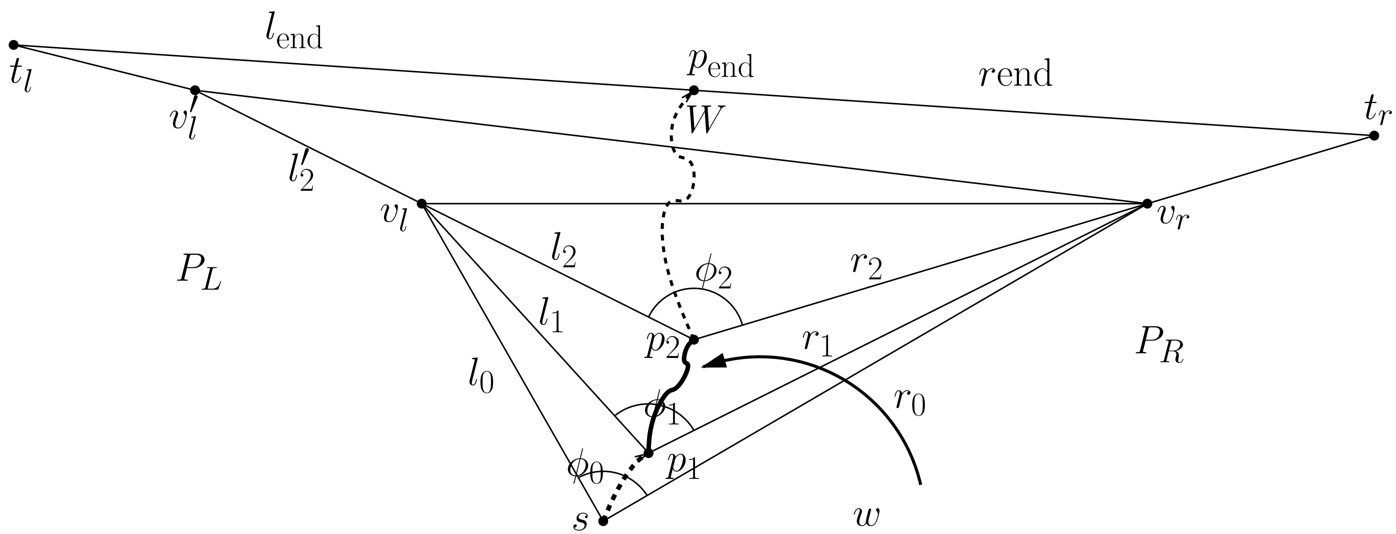
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- Guarantee: $\frac{|w|+K_{\phi_2}\cdot\ell_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w|+K_{\phi_2}\cdot r_2}{r_1} \leq K_{\phi_1}$
- Combine, single condition for w
- $|w| \leq \min\{\, K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \,,\, K_{\phi_1}r_1 - K_{\phi_2}r_2 \,\}$
- Change of a vertex at p_2? Remains guilty!



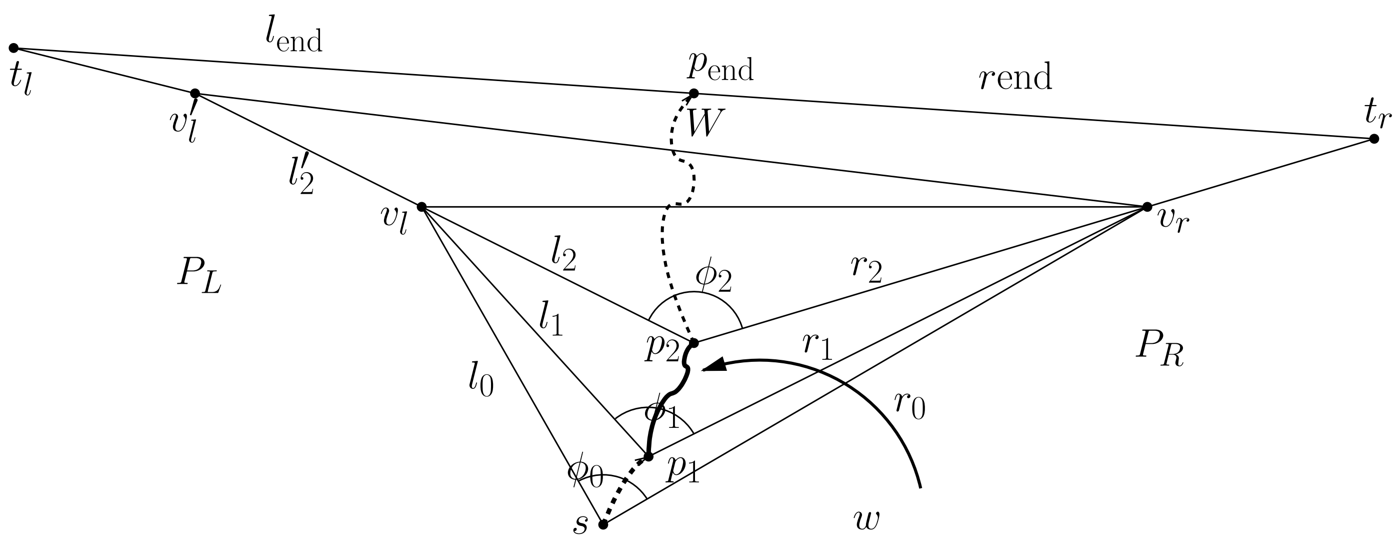
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- Change left hand: Condition
  $|w| \leq \min\{\, K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \,,\; K_{\phi_1}r_1 - K_{\phi_2}r_2 \,\}$
- There is opt. strategy for ϕ_2
- Show: $\frac{|w|+K_{\phi_2}\cdot(\ell_2+\ell_2')}{(l_1+\ell_2')} \leq K_{\phi_1}$



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

$$
\begin{aligned}
|w| &\leq K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \\
&= K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 + K_{\phi_2}\ell_2' - K_{\phi_2}\ell_2' \\
&\leq K_{\phi_1}(\ell_1 + \ell_2') - K_{\phi_2}(\ell_2 + \ell_2')
\end{aligned}
$$

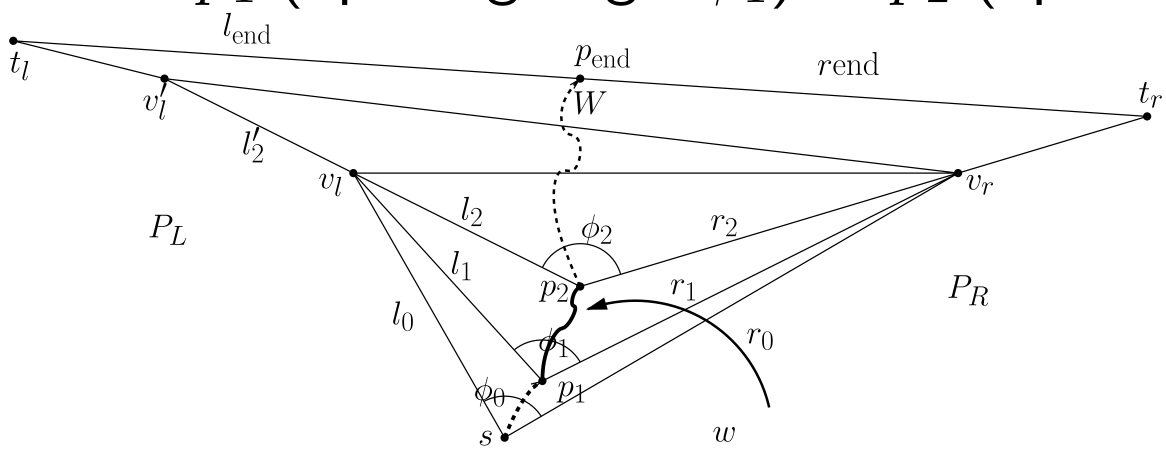


Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

Lemma Let S be a strategy for funnels with opening angles $\phi_2 \geq \frac{\pi}{2}$ and competitive ratio K_{ϕ_2}. We can extend this strategy to a strategy with ratio K_{ϕ_1} for funnels with opening angles ϕ_1 where $\phi_2 > \phi_1 \geq \frac{\pi}{2}$, if we guarantee

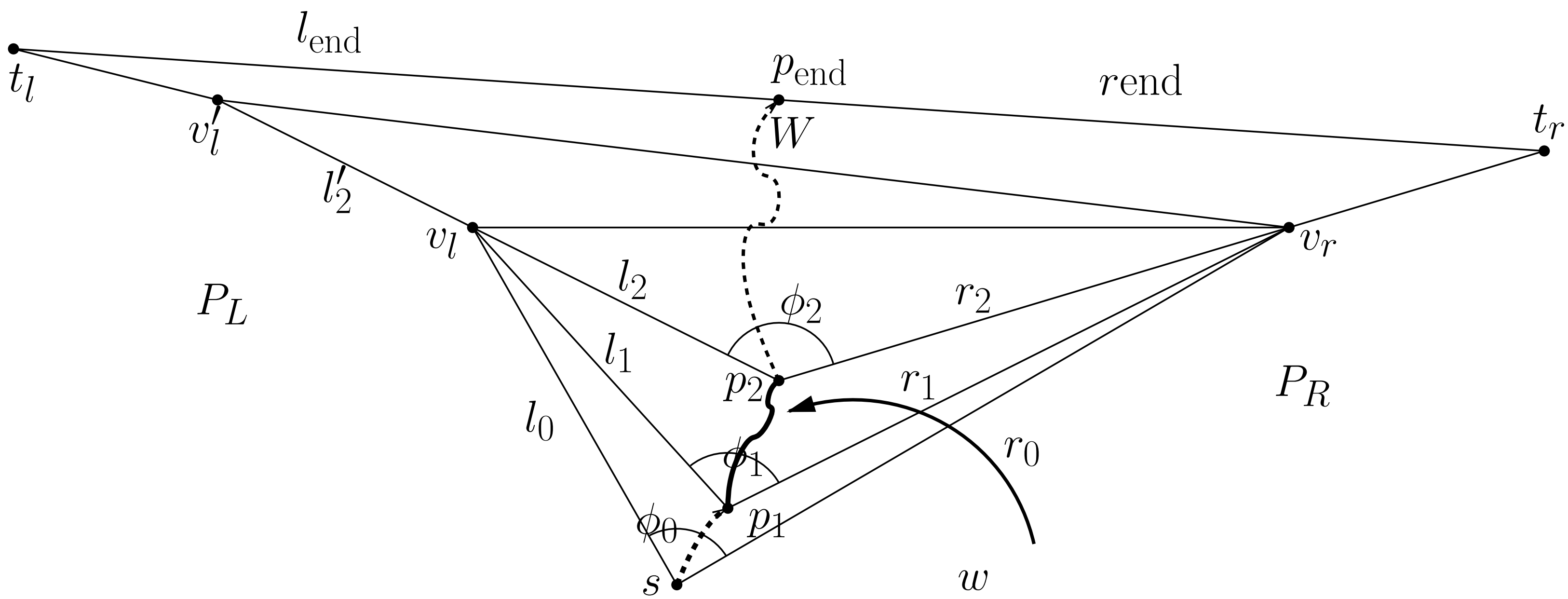
$$|w| \leq \min\{\, K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \;,\; K_{\phi_1}r_1 - K_{\phi_2}r_2 \,\}$$

for the path w from p_1 (opening angle ϕ_1) to p_2 (opening angle ϕ_2).



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- If $|w| \leq \min\{ K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \, , \; K_{\phi_1}r_1 - K_{\phi_2}r_2 \}$ holds, then
- $|W| \leq \min\{ K_{\phi_0} \cdot |P_L| - K_\pi \ell_{\text{End}} \, , \; K_{\phi_0} \cdot |P_R| - K_\pi r_{\text{End}} \}$.

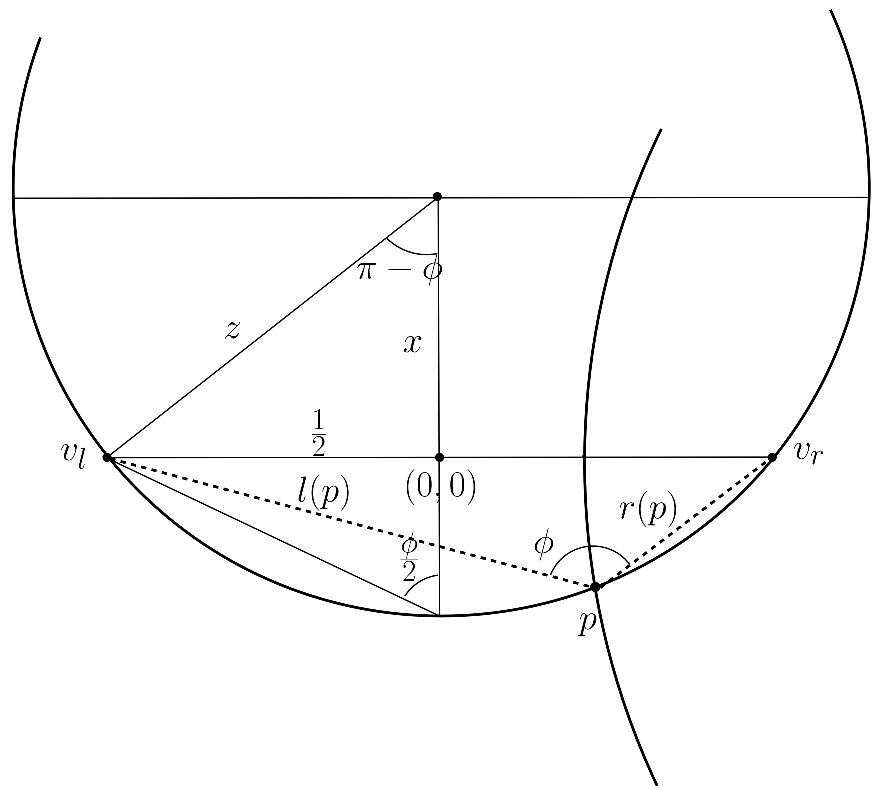


Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- $|w| \leq \min\{ K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \, , \, K_{\phi_1}r_1 - K_{\phi_2}r_2 \}$
- How to fulfil this?
- Equality for both sides: $K_{\phi_2}(\ell_2 - r_2) = K_{\phi_1}(\ell_1 - r_1)$
- Good choice for both sides!
- Defines a curve!
- We start with $A = K_{\phi_0}(\ell_0 - r_0)$
- Parametrisation!

$$A = K_{\phi_0}(\ell_0 - r_0)$$

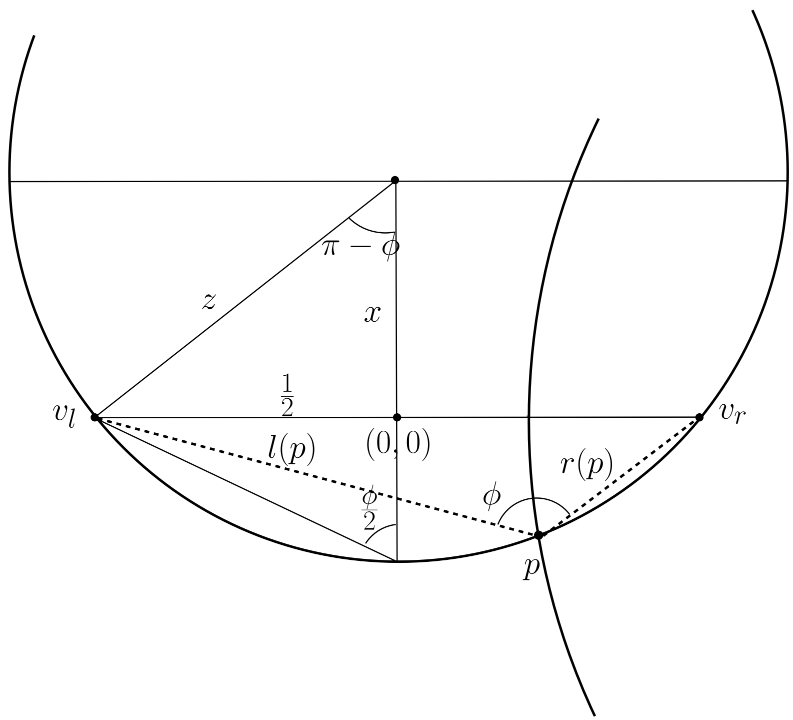
- Hyperbola: $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, $l - r = 2a$, $2c$, $a^2 + b^2 = c^2$
- Circle: $X^2 + (Y - x)^2 = z^2$, $r = z$, $(0, x)$



Intersection with circle and hyperbola

- Hyperbola: $\frac{X^2}{\left(\frac{A}{2K_\phi}\right)^2} - \frac{Y^2}{\left(\frac{1}{2}\right)^2 - \left(\frac{A}{2K_\phi}\right)^2} = 1$

- Circle: $X^2 + \left(Y + \frac{\cot\phi}{2}\right)^2 = \frac{1}{4\sin^2\phi}$



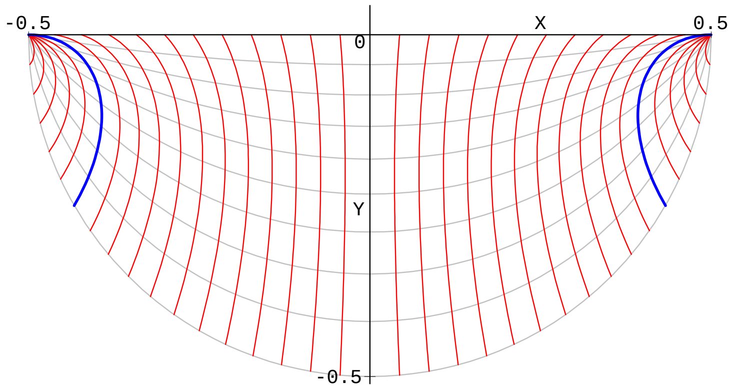
Opt. strat. for opening angle $\pi \geq \varphi_0 \geq \pi/2$!

Intersection: Verification by insertion!

$$X(\phi) = \frac{A}{2} \cdot \frac{\cot\frac{\phi}{2}}{1+\sin\phi} \cdot \sqrt{\left(1+\tan\frac{\phi}{2}\right)^2 - A^2}$$

$$Y(\phi) = \frac{1}{2} \cdot \cot\frac{\phi}{2} \cdot \left(\frac{A^2}{1+\sin\phi} - 1\right)$$

where $A = K_{\phi_0}(\ell_0 - r_0)$

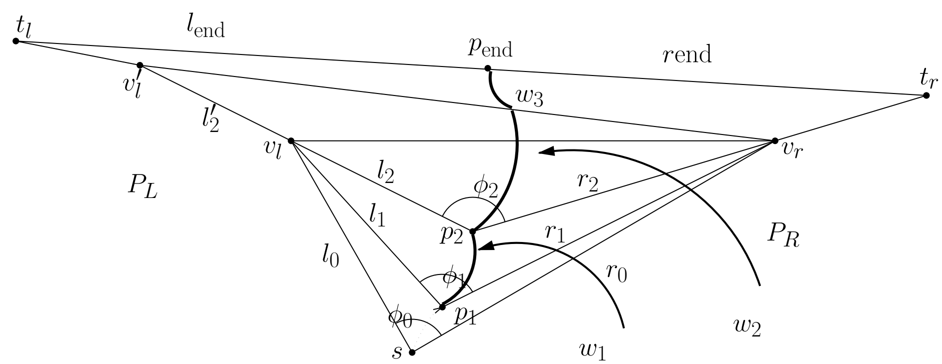


Opt. strat. for opening angle $\pi \geq \varphi_0 \geq \pi/2$!

$$X(\phi) = \frac{A}{2} \cdot \frac{\cot\frac{\phi}{2}}{1+\sin\phi} \cdot \sqrt{\left(1+\tan\frac{\phi}{2}\right)^2 - A^2}$$

$$Y(\phi) = \frac{1}{2} \cdot \cot\frac{\phi}{2} \cdot \left(\frac{A^2}{1+\sin\phi} - 1\right)$$

Change of the boundary points. A also changes, new piece of curve!



Opt. strat. for opening angle $\pi \geq \varphi_0 \geq \pi/2$!

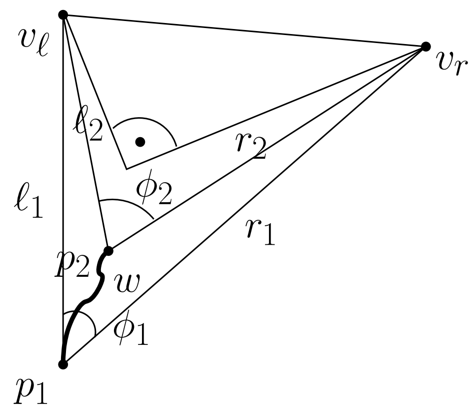
Theorem: The goal of a funnel with opening angle $\phi_0 > \frac{\pi}{2}$ can be found with ratio K_{ϕ_0}.

Proof: Show that the curves fulfil:
$|w| \leq \min\{ K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \, , \, K_{\phi_1}r_1 - K_{\phi_2}r_2 \}$

For any small piece w of the curve. Analytically, lengthy proof! Experiementally!

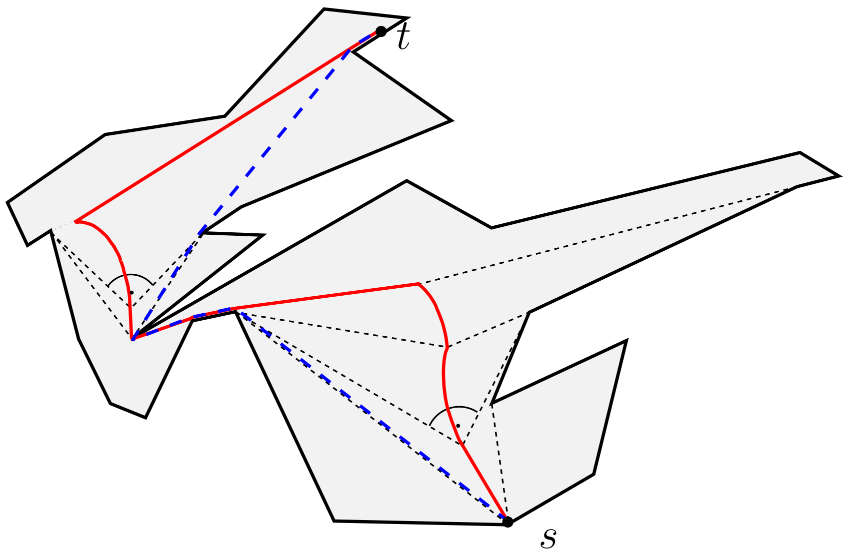
Opt. strat. opening angle $0 \leq \varphi_0 \leq \pi/2$!

- The same approach
- But independent from the angle
- Dominated by *factor* $K_{\pi/2} = \sqrt{2}$
- Require: $w \leq \min\{\sqrt{2}(\ell_1 - \ell_2)\ ,\ \sqrt{2}(r_1 - r_2)\}$.
- Equality: $\ell_1 - \ell_2 = r_1 - r_2$
- Current angular bisector: Hyberbola!



Opt. strat. opening angle $0 \leq \varphi_0 \leq \pi$!

Combine strategy 1 and strategy 2



Theorem: In an unknown street-polygon beginning from the source s we can find the target t with an optimal online strategy with competitive ratio $\sqrt{2}$.

Optimal strategy "Worst-Case-Aware"

As long as target t is not visible:

Compute current v_ℓ and v_r.

If only one exists: Move directly toward the other.

Otherwise. Repeat:

New reflex vertex v_ℓ' or v_r' is detected:

Use v_ℓ' or v_r' instead of v_ℓ or v_r.

Let ϕ be the angle between v_ℓ, the current poyition and v_r.

If $\phi \leq \frac{\pi}{2}$: Follow the current angular bisctor!

If $\phi > \frac{\pi}{2}$: Follow the curve $(X(\phi), Y(\phi))$.

Until either v_ℓ or v_r is explored.

Move toward the non-explored vertex.

Move toward the goal.