Multi-list Traversal Strategies (cf. [1])

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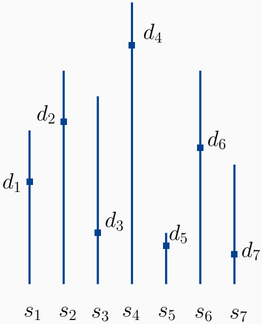
The multi-list traversal problem (MLTP)



Definition (traversal costs)

$\Lambda = \{\lambda_1, \lambda_2, \ldots \lambda_m\}$ lists of length $\lambda_i \in \mathbb{N}$.
For ordering $(\lambda_1, \lambda_2, \ldots \lambda_m)$ of Λ, the traversal costs of a strategy $\mathcal{S}$ are:

$$\mathsf{TC}(\mathcal{S}, (\lambda_1, \lambda_2, \ldots \lambda_m)) = \sum_{1 \leq i \leq m} d_i,$$

$\mathcal{S}$ traversed ith list up to depth $d_i \leq \lambda_i$.

Definition (multi-list traversal problem - MLTP)

Given:	Set Λ of m lists, each of unknown length.
Aim:	Reach end of one list (with small traversal costs).

Note: no costs for switching lists; traversal of a list can be continued.

Applying an alternative cost measure

1. Consider partially informed variant of MLTP

    Find reasonable strategy. (fixed depth traversal FDT)

    Define cost measure. ($\xi_\Lambda, \overline{\xi}_\Lambda$)

    Justification of the strategy/cost measure.

2. Reconsider uninformed variant of MLTP

    Suggest online strategy. (hyperbolic traversal HT)

    Prove competitiveness w.r.t. new cost measures.

Consider partially informed variant of MLTP

A simple and reasonable strategy: fixed depth traversal (FDT)

Given: Set Λ of m lists of known length, but unknown ordering.
Aim: Reach end of one list with small traversal costs.

$\Longrightarrow$ Lower bound for traversal costs is $\min_{1 \leq i \leq m} \lambda_i$.
$\Longrightarrow$ Any strategy that traverses every list up to depth $d \geq \min_{1 \leq i \leq m} \lambda_i$ is successful.

Fixed-depth-traversal

Input: Set Λ of m lists, fixed depth $d \in \mathbb{N}_0$

for i from 1 to m **do**
  traverse list λ_i up to depth d;
end for

The alternative cost measure - worst case

Definition (intrinsic maximum traversal costs)

The maximum traversal costs are defined as

$$\mathsf{MTC}_\Lambda(\mathsf{FDT}(d)) := \max_{\pi \in S_m} \mathsf{TC}(\mathsf{FDT}(d), \pi(\Lambda)).$$

The intrinsic maximum traversal costs are defined as

$$\xi_\Lambda := \min_{1 \leq k \leq m} \mathsf{MTC}_\Lambda(\mathsf{FDT}(\lambda_k)).$$

Theorem (cf. [1], Theorem 1)

Reorder s.th. $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$, then

$$\xi_\Lambda = \min_{1 \leq i \leq m} i \cdot \lambda_i$$

$$i_\Lambda := \operatorname*{argmin}_{1 \leq i \leq m} i \cdot \lambda_i$$

$\rightsquigarrow$ Best FDT-strategy for Λ in the worst case.

The alternative cost measure - average case

Definition (intrinsic average traversal costs)

The average traversal costs are defined as

$$\mathsf{ATC}_\Lambda(\mathsf{FDT}(\lambda_k)) := \operatorname*{avg}_{\pi \in S_m} \mathsf{TC}(\mathsf{FDT}(\lambda_k), \pi(\Lambda)).$$

The intrinsic average traversal costs are defined as

$$\overline{\xi}_\Lambda := \min_{1 \leq k \leq m} \mathsf{ATC}_\Lambda(\mathsf{FDT}(\lambda_k)).$$

Theorem (cf. [1], Lemma 1)

Reorder s.th. $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$, *then*

$$\overline{\xi}_\Lambda = \min_{1 \leq i \leq m} \frac{(m+1) \cdot \lambda_i}{m - i + 2}$$

$$\overline{i}_\Lambda := \operatorname*{argmin}_{1 \leq i \leq m} \tfrac{\lambda_i}{m-i+2}$$

$\rightsquigarrow$ Best FDT-strategy for Λ in the average case.

Justification of FDT

1. The competitive ratio of breadth-first traversal ($=$ FDT(λ_m)) is $\Omega(m)$ and the competitive ratio of depth-first traversal ($=$ FDT(λ_1)) is unbounded. [1]
2. No traversal strategy that is successful on all permutations of Λ, has fewer traversal costs than ξ_Λ in the worst case. [2]
3. Any traversal strategy that terminates with traversal costs of at most $\overline{\xi}_\Lambda/3$ on all presentations of Λ, fails with probability $1/2$ on a random presentation of Λ. [3]
   $\rightsquigarrow$ $\overline{\xi}_\Lambda$ is $\theta\left(\frac{(m+1)\cdot\lambda_{\overline{i}_\Lambda}}{m-\overline{i}_\Lambda+2}\right)$

[1] cf. [1], Theorem 3
[2] cf. [1], Proof of Theorem 1
[3] cf. [1], Lemma 2 and Theorem 2

Reconsider uninformed variant of MLTP

Hyperbolic traversal (HT)

Given: Set Λ of m lists of unknown length.

Aim: Reach end of one list with small traversal costs.

Hyperbolic-traversal

Input: List Λ

$c \leftarrow 1$;
while no list fully explored **do**
    for i from 1 to m **do**
        explore list i up to depth $\lfloor \frac{c}{i} \rfloor$;
    end for
    $c \leftarrow c + 1$;
end while

Competitiveness of HT

Theorem

HT solves MLTP with $O\left(\xi_\Lambda \cdot \ln(\min(m, \xi_\Lambda))\right)$ maximum traversal costs. [4]

Theorem

HT solves the MLTP with $O\left(\overline{\xi}_\Lambda \cdot \ln(\min(m, \overline{\xi}_\Lambda))\right)$ in the average traversal costs. [5]

Optimality

As D. Kirkpatrick shows in [1], HT is also optimal w.r.t. the alternative cost measure.

[4] cf. [1], Theorem 4
[5] cf. [1], Theorem 6

References I

D. G. Kirkpatrick.
Hyperbolic dovetailing.
In *Algorithms - ESA 2009, 17th Annual European Symposium, Copenhagen, Denmark, September 7-9, 2009. Proceedings*, pages 516--527, 2009.