The escape path problem (cf. [1])

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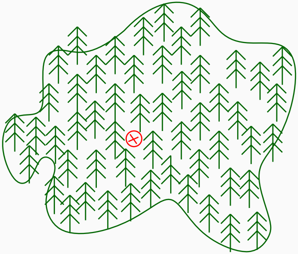
The escape path problem (EPP)



Definition (escape path/costs)

Polygon P. An escape path for s $\in P$ is a continuous and rectifiable mapping $\gamma : [0, b] \to \mathbb{R}^2$, s.th.

- $\gamma(0) = \mathrm{s}$,
- $\exists e \in [0, b] : \ \gamma(e) \in \partial P$ and $\forall t < e$ $\gamma(t) \in P \setminus \partial P$.

The escape costs for γ starting in s are defined as $\mathrm{EC}(\gamma, \mathrm{s}) := L(\gamma|[0, e])$.

Definition (escape path problem - EPP)

Given: A polygon P and a starting point s $\in P$.

Aim: Reach ∂P with small escape costs.

Applying an alternative cost measure

1. Consider partially informed variant of EPP

   Find reasonable strategy. (certificate path for distance x)

   Define cost measure. (Π_s)

   Justification of the strategy/cost measure.

2. Reconsider uninformed variant of EPP

   Suggest online strategy. (Spiral-escape)

   Prove competitiveness w.r.t. new cost measures.

Consider partially informed variant of EPP

A simple and reasonable strategy: certificate path $\Pi_s(x)$

Given: An unknown P and an unknown $s \in P$, but exact distance from s to ∂P in all directions.

Aim: Reach ∂P with small escape costs.

$\implies$ Lower bound for escape costs is $d := \min_{p \in \partial P} \overline{sp}$.

$\implies$ Any straight path of length $D := \max_{p \in \partial P} \overline{sp}$ is an escape path.

Definition (Certificate path for distance x)

The *certificate (escape) path for distance* x consists of

- a straight line segment of length x starting in s and
- an arc segment of length α around s with radius x.

The new cost measure

Definition (Overall certificate path)

The *maximum arc segment* $\alpha_s(x)$ for the certificate path for distance x is defined as the longest arc segment of a circle $C_s(x)$ that fully lies inside P. If $C_s(x) \subset P \setminus \partial P$, we set $\alpha_s(x) = \infty$.

The overall certificate path Π_s is defined as the shortest certificate path over all distances x. Thus, the overall escape costs (length of Π_s) is

$$\Pi_s := \min_{x \in \mathbb{R}_{\geq 0}} x \cdot (1 + \alpha_s(x)) \,.$$

Justification of Π_s

1. The certificate path only depends on *P* and s, not on the orientation of *P*.
2. It is an intuitive strategy that balances BFS and DFS. The competitive ratio of breadth-first search $(= \Pi_\mathsf{s}(d))$ is $O(1)$ and the competitive ratio of depth-first search $(= \Pi_\mathsf{s}(D))$ is unbounded.
3. For any environment where *ultimate optimal escape paths* are known, the certificate path has always fewer escape costs in the worst case.

Reconsider uninformed variant of EPP

Spiral-escape

Given: An unknown starting point s lies in the kernel of a polygon P of unknown shape.

Aim: Reach ∂P with small escape costs.

General spiral strategy $\mathcal{S} : \mathbb{R} \to \mathbb{R}^2$ with $\varphi \mapsto (\varphi, a \cdot e^{\varphi \cot(\beta)})$.

$\rightarrow$

Optimize against two extreme cases, i.e. choose β s.th. both cases attain the same ratio.

Spiral-escape

Choose a direction and follow the logarithmic spiral with eccentricity $\beta = \operatorname{arccot}\left(\frac{\ln(2\pi+1)}{2\pi}\right)$.

Competitiveness of Spiral-escape

Theorem (Upper Bound)

Spiral-escape solves EPP with a competitive ratio < 3.318674.

Theorem (General Lower Bound)

Any online strategy solves EPP with a competitive ratio ≥ 3.313126.

Optimality

Spiral-escape is (almost) optimal w.r.t. the alternative cost measure.

References I

E. Langetepe and D. Kübel.
Optimal online escape path against a certificate.
CoRR, abs/1604.05972, 2016.