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Randomized Algorithms and
Probabilistic Analysis
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Problem Set 6

Problem 1

Give examples for the following situations or reason that they cannot occur.

1. Draw paths $P(i)$ and $P(j)$ and give a value t such that $\text{offset}(t, j) = 2|P(i)|$.
2. Draw a path $P(i)$ and possibly other intersecting paths and give a value t such that $\text{offset}(t, i) = 2|P(i)|$.
3. Draw a path $P(i)$ with $|P(i)| = 4$ and a path $P(j)$ and give a value L such that $\text{lucky}(L + 1, L) = \emptyset$, $\text{lucky}(t, L) = \{j\}$ for j at all times $t \in \{L + 2, \dots, |P(i)| + L\}$.
4. Draw a path $P(i)$ and possibly other intersecting paths and give a value for L such that $\text{lucky}(t, L) \neq \text{lucky}(t', L)$ for all $t, t' \in \{L + 1, \dots, |P(i)| + L\}$ with $t \neq t'$.
5. Draw paths $P(i)$ and $P(j)$ and give values L, t, t' with $t < t'$ and $t' \leq |P(i)| + L - 1$ such that $\text{lucky}(t, L) \neq \emptyset$ and $\text{lucky}(t', L) = \emptyset$.
6. Draw paths $P(i)$, $P(j)$ and possibly other paths and give values t and L such that $\text{lucky}(t, L) = \{j\}$, $\text{lucky}(t + 1, L) = \emptyset$ and $\text{lucky}(t + 2, L) = \{j\}$.

Problem 2

Assume that we throw $m = \lfloor 5n \log_2 n \rfloor$ balls into n baskets. For each ball, the basket is chosen uniformly at random. Let X_i be the number of balls in basket i for $i \in \{1, \dots, n\}$, and set $X = \max_{i=1,\dots,n} X_i$. Show that

$$\mathbf{Pr}(X \geq 30 \log_2 n) \leq \frac{1}{n^c}$$

holds for a suitable constant $c > 0$.

Problem 3

Show that the recurrence

$$h_0 = 1 + h_1, \quad h_n = 0, \quad \forall j \in \{0, \dots, n - 1\} : h_j \leq 1 + \frac{2}{3} h_{j-1} + \frac{1}{3} h_{j+1}$$

implies that $h_j \leq 2^{n+2} - 2^{j+2} - 3(n - j)$ for all $j \in \{0, \dots, n\}$. *Hint:* First show by induction that for all $j \in \{0, \dots, n - 1\}$, $h_j \leq h_{j+1} + 2^{j+2} - 3$ holds.

(This task completes the proof of Lemma 4.3).