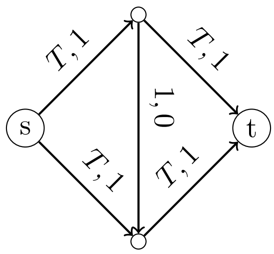
Institut für Informatik
Prof. Dr. Heiko Röglin
Dr. Melanie Schmidt

Randomized Algorithms and
Probabilistic Analysis
Summer 2016

Problem Set 12

Problem 1

Recall the following instance from problem set 11.



1. Give a short proof that choosing the costs of all five edges according to independent density functions $f_e : [0, 1] \to [0, \phi]$ implies that the SSP algorithm converges in a constant number of steps for any integer T. Only use Property 8.9.
2. Give an even shorter proof that uses Property 8.9 and Corollary 8.3.
3. Extend your proof from 2. to arbitrary input graphs of constant size.

Problem 2

Let $G = (V, E)$ be an undirected complete graph and let $c : E \to [0, 1]$ be a cost function that assigns a cost c_e to each $e \in E$. We consider the following problem, which we call *rooted k-MST*: Given G, a vertex $r \in V$ and a number $k \in \{1, \ldots, |V|\}$, find a tree in G that spans exactly k vertices, including r, and has minimal cost.

Let $T = (V, E')$ be a tree in G. A pair of edges $\{e, f\}$ with $e \in E', f \notin E'$ is an *improving pair* iff $T \cup \{f\} \setminus \{e\}$ is a tree that contains r and $\Delta(e, f) := c_e - c_f > 0$. Consider the following algorithm for the rooted k-MST problem:

1. Start with an arbitrary tree T that spans k vertices, including r.
2. While an improving pair $\{e, f\}$ exists,

   2.1 remove e from T, add f to T.
3. Output T.

Assume that all c_e are ϕ-perturbed numbers (from $[0, 1]$). Analyze the expected number of iterations of this algorithm similarly to Theorem 9.4.