Online Motion Planning MA-INF 1314
Application Search Path Approx.!

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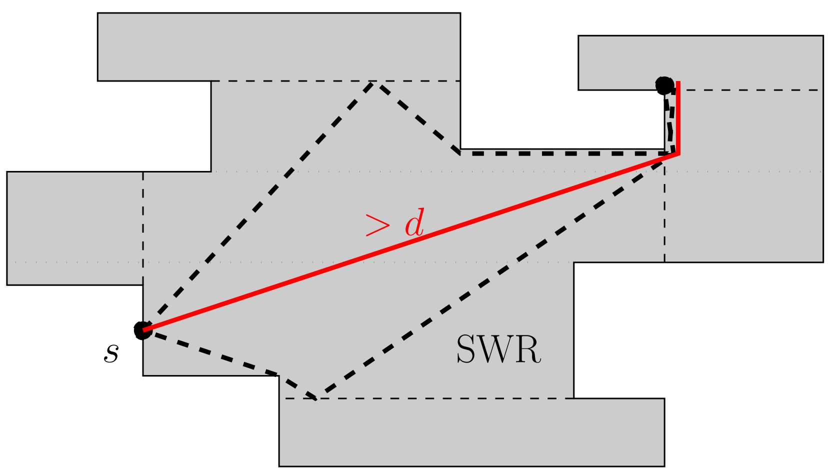
Rep.: Search Path Approx. Applications (vision)!

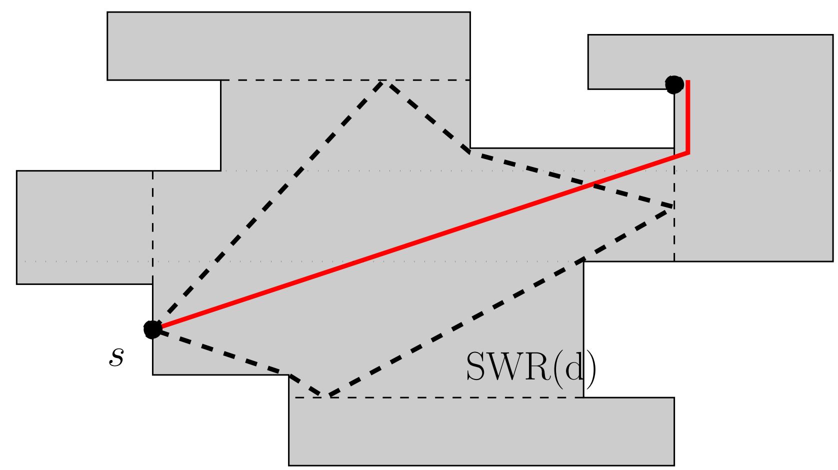
- Simple polygon, Offline: SWR ($C_\beta = 1 = \beta$)
  $\Rightarrow$ 8-Approximation
- Rectilinear Polygons, Online: Greedy-Online ($C_\beta = \sqrt{2}, \beta = 1$)
  $\Rightarrow$ $8\sqrt{2}$-Approximation
- Simple Polygons, Offline: Polytime Alg. ($C_\beta = 1, \beta = 1$)
  $\Rightarrow$ 8-Approximation
- Simple Polygons, Online: PolyExplore ($C_\beta = 26, \beta = 1$)
  $\Rightarrow$ 212-Approximation

Consider exploration task! Full and depth restricted!

Rep.: SWR (Rect. polygon) offline depth restriction

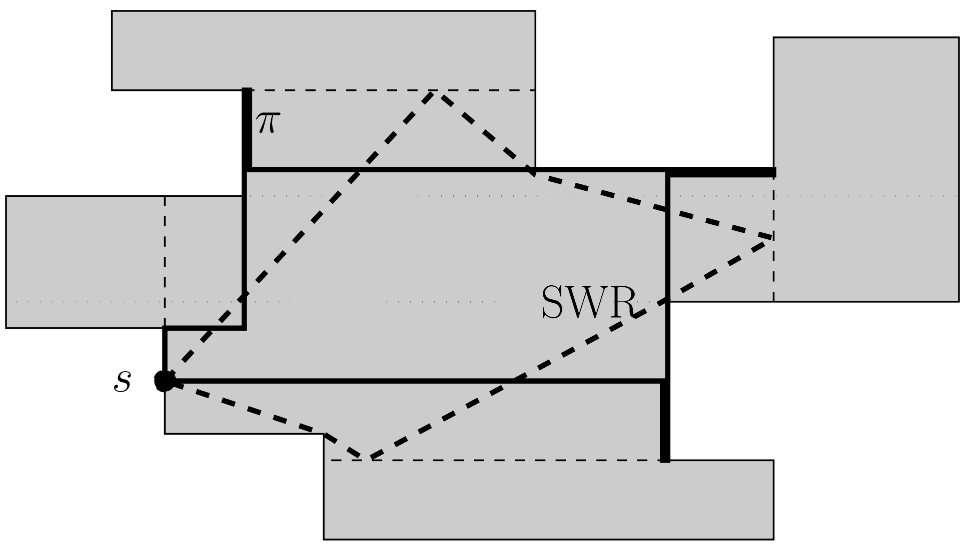
- Ignore cuts with distance $> d$, Shortest path to cut
- Ignore a cut here, optimal algorithm
- $\mathrm{Expl}\,(d) = \mathrm{Expl}_{\mathrm{OPT}}(d)$
- **Theorem**: 8 Approximation of optimal search path!





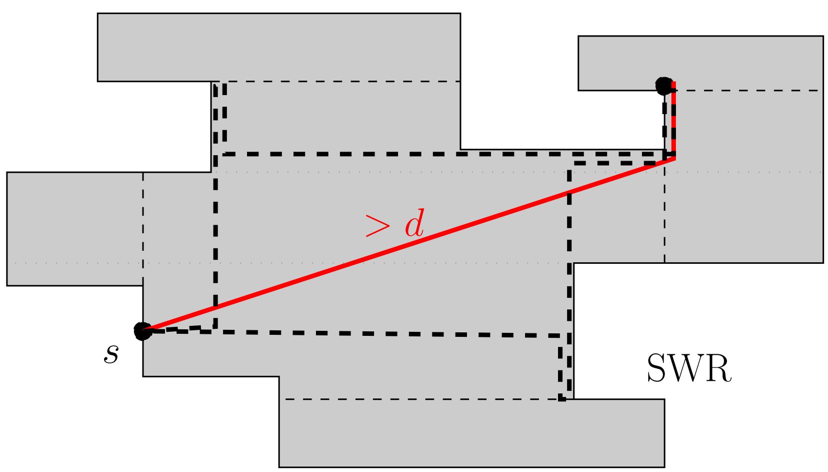
Rep.: Rectilinear polygons Online

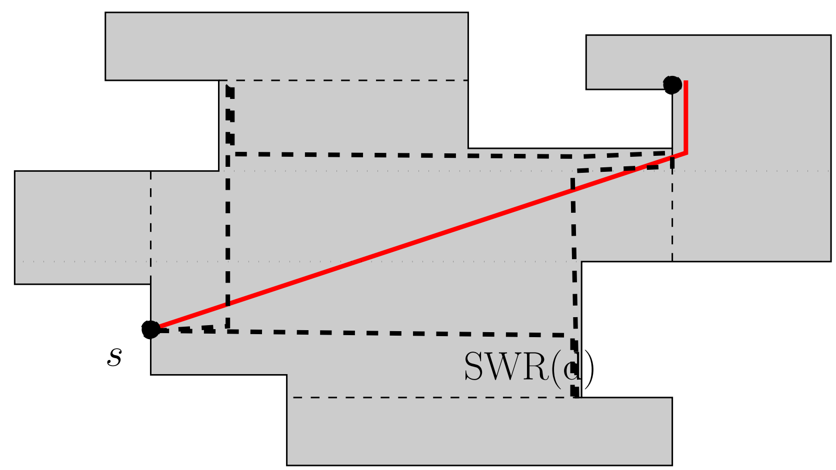
- Assume, s boundary point
- Greedy! Scan the boundary up to the first invisible point. Move to the cut on the shortest path!
- Shortest L_1-path to the cut, online!
- **Algorithmus** Always approach the next reflex vertex along the boundary that blocks the visibility.



Rep.: L_1-opt./$\sqrt{2}$-competitive! Theorem

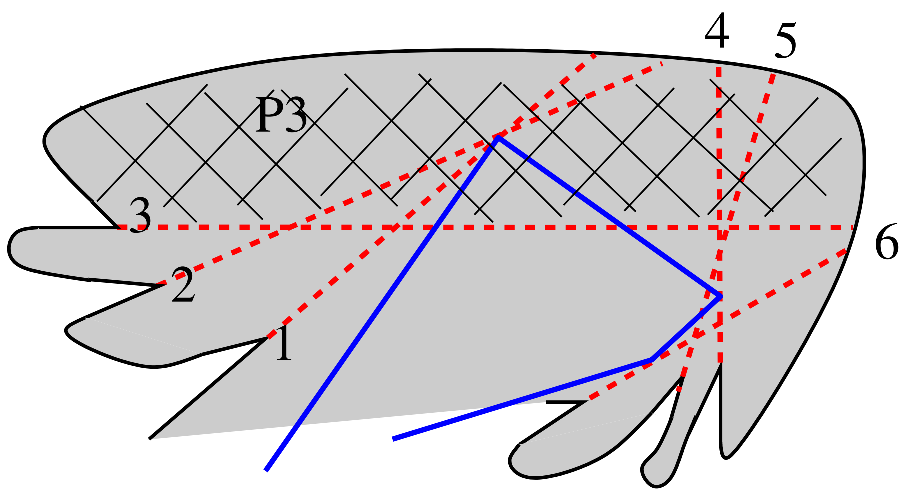
- $\sqrt{2}$-competitive
- Depth restrictable!
- Online: Ignore Cuts with distance $> d$
- $\mathrm{Expl}_{\mathrm{ONL}}(d) \leq \sqrt{2}\ \mathrm{Expl}_{\mathrm{OPT}}(d)$
- **Theorem**: $8\sqrt{2}$-Approximation





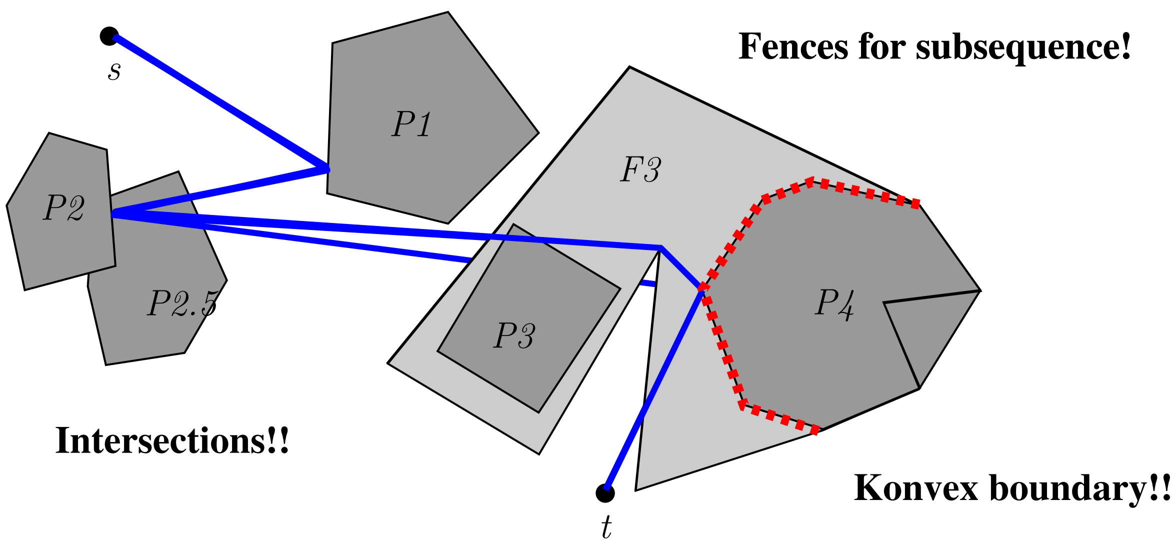
SWR (General case): Offline!

- Corner problem!!
- Sequence of essential cuts, successive cuts
- Not visited by order along boundary.
- But the corresponding P_{c_i}!!!

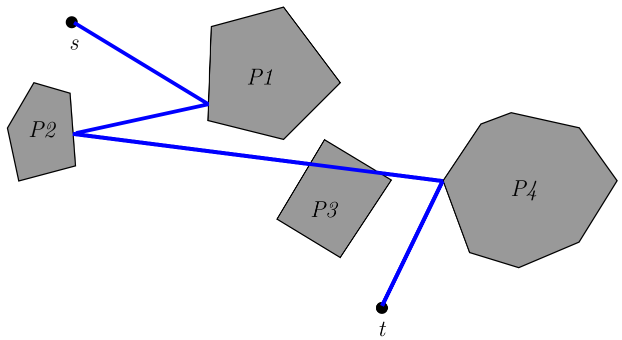


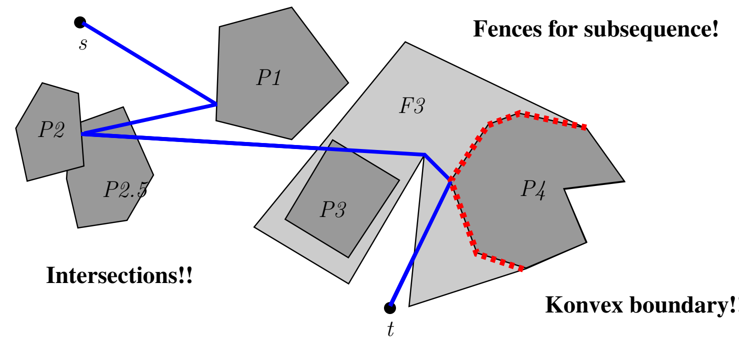
Touring a sequence of polygons (TPP)

- Sequence of convex polygons
- Start s, target t
- Visit polygons w.r.t. sequence, shortest path



TPP



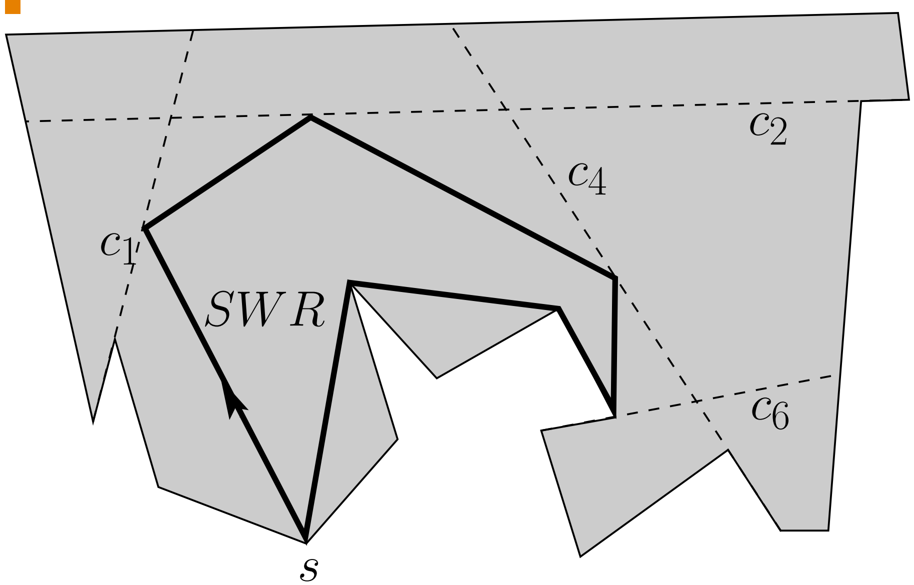


- Simple version:
- $O(nk \log \frac{n}{k})$
- Build(Query): $O(nk \log \frac{n}{k})$
- Compl.: $O(n)$
- Query (fixed s): $O(k \log \frac{n}{k})$

- General version:
- Fences, convex boundary, etc.
- $O(nk^2 \log n)$
- Build(Query): $O(nk^2 \log n)$
- Compl.: $O(nk)$
- Query (fixed s): $O(k \log n + m)$

Results from: Dror, Efrat, Lubiw, Mitchell 2003!!

Application: SWR



Essential *parts*!

Use the order along the boundary!

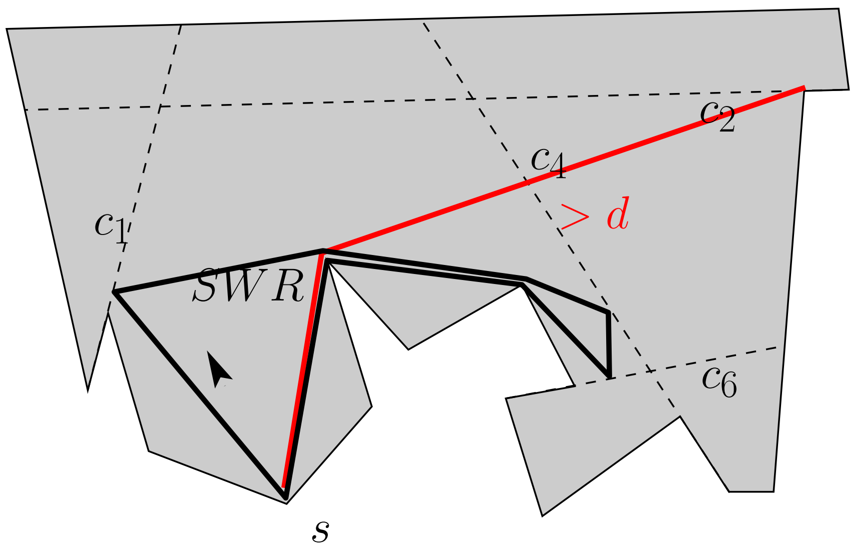
One common fence, intersections!

Start and target identical!

- $O(n^4)$ '91
- $O(n^4)$ Tan et al. '99
- $O(n^3 \log n)$ by this result!
- **Theorem**

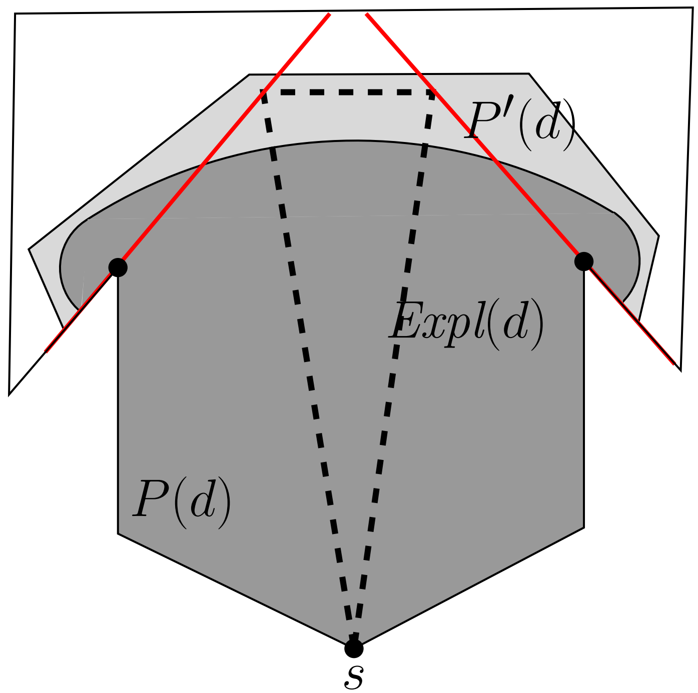
Application: General simple polygons Offline

- Compute optimal exploration tour
- Agent with vision, start s at the boundary
- Depth restriction: Ignore cuts with distance $> d$
- $\text{Expl}\,(d) = \text{Expl}_{\text{OPT}}(d)$
- **Theorem**: 8 Approximation, Online 212 (postponed)



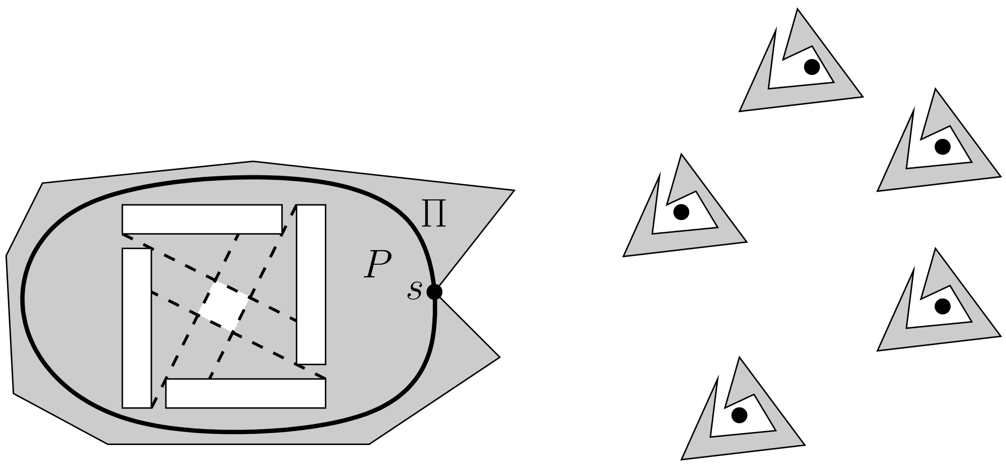
Remark: Depth restriction Offline

- $P(d)$ subset of P
- $\mathrm{Expl}\,(d) = \mathrm{Expl}_{\mathrm{OPT}}(d)$ can leave $P(d)$



Vision: Negative result, polygon with *holes*

- Much more difficult
- Example: See boundary ⇔ see everything
- Not true for such scenes
- Offline: Computation SWR is NP-hart, reduction idea TSP



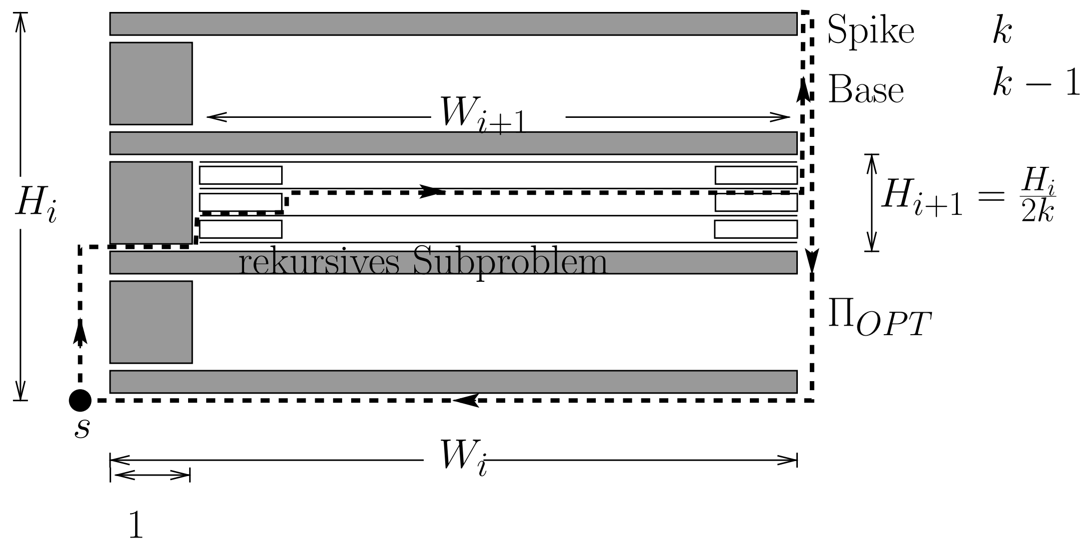
Polygons with holes

There is no constant online approximation of the optimal search ratio

Theorem Let A be an online strategy for the exploration of a polygon with n obstacles (holes), we have: $|\Pi_A| \geq \sqrt{n}|\Pi_{OPT}|$

Proof: LB by examples!

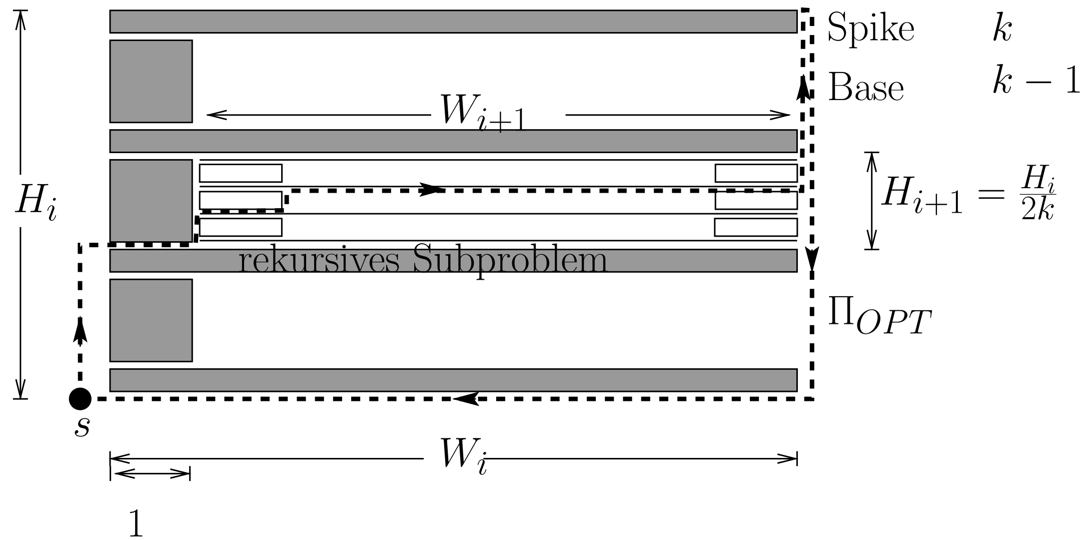
Polygon with holes: $|\Pi_A| \geq \sqrt{n}|\Pi_{OPT}|$



- $W_1 = 2k$, $H_1 = k$, k spikes, $(k-1)$ bases, $(2k-1)k$ rectangles
- $H_i = \frac{H_1}{(2k)^{i-1}}$, $W_i = 2k - i + 1 \geq k$, $i = 1, \ldots, k$
- Situation H_i: Online strategy does not know position of block H_{i+1}

- Recursively
- Left side: Look behind any block
- Right side: Move once upwards
- Adversary: Find block after $\Omega(k)$ steps
- Altogether: $\Omega(k \times k)$ for any strategy

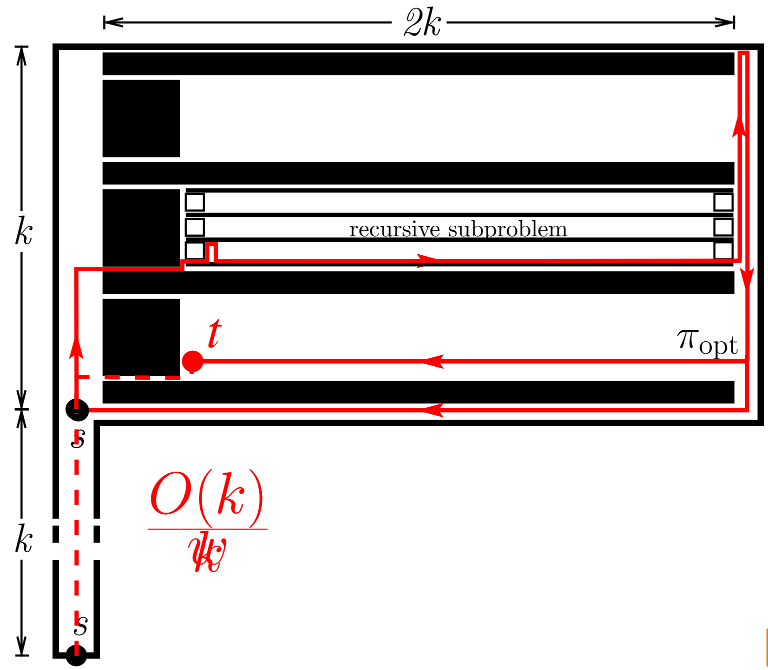
Polygons with holes: $|\Pi_A| \geq \sqrt{n}|\Pi_{OPT}|$



- Optimal strategy: Move directly to the block
- Go on recursively, at the end move along any block
- $[\Pi_{OPT}| \approx 2W_1 + 2k < 7k$
- $k = \lfloor \sqrt{n} \rfloor$ gives the result

Polygons with holes Corollary

- No $O(1)$-competitive exploration for such environments ($\Omega(\sqrt{n})$)
- Optimal exploration has a bad Search Ratio
- Trick: Extension
- Then: Optimal exploration has Search Ratio $O(1)$
- Any online strategy has Search Ratio $\Omega(k)$



Summary

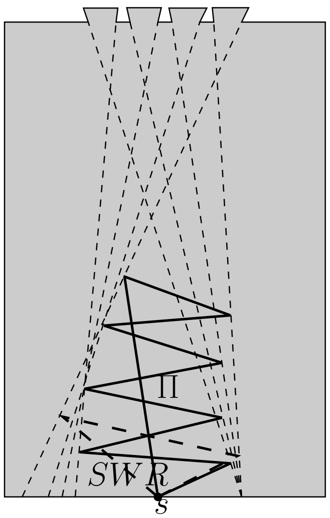
- Connection between exploration and search:

- $\exists$ constant-competitive, depth-restrictable exploration strategy
  $\Rightarrow \exists$ search strategy with competitive Search Ratio

- $\nexists$ constant-competitive exploration strategy,
  but $\exists$ 'extendable' lower bound
  $\Rightarrow \nexists$ search strategy with competitive Search Ratio

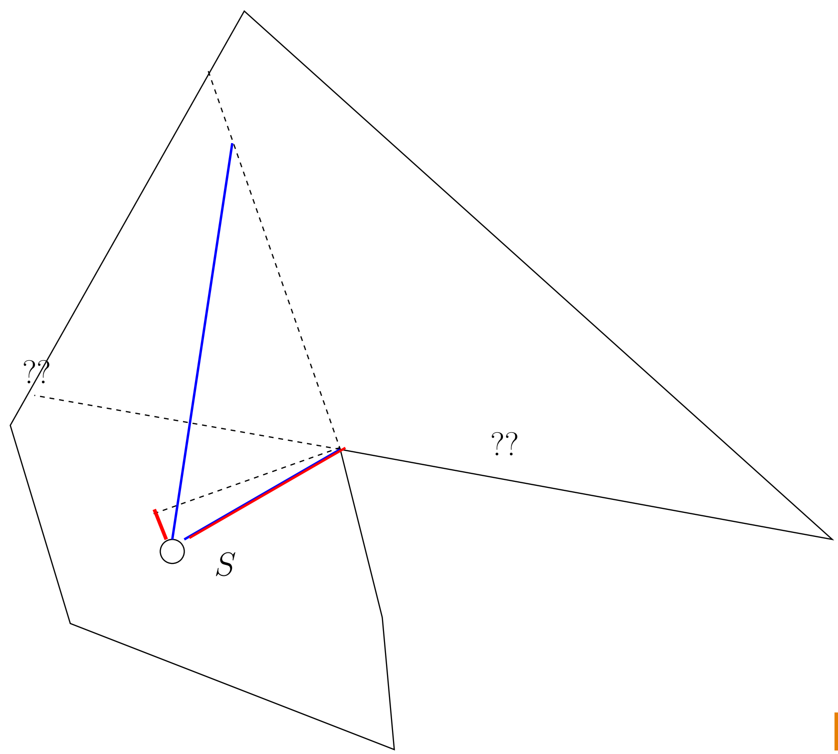
Online exploration arbitrary polygon

- Essential cuts, appearance order along the boundary
- Not competitive
- Sudivision: Right and left reflex vertices



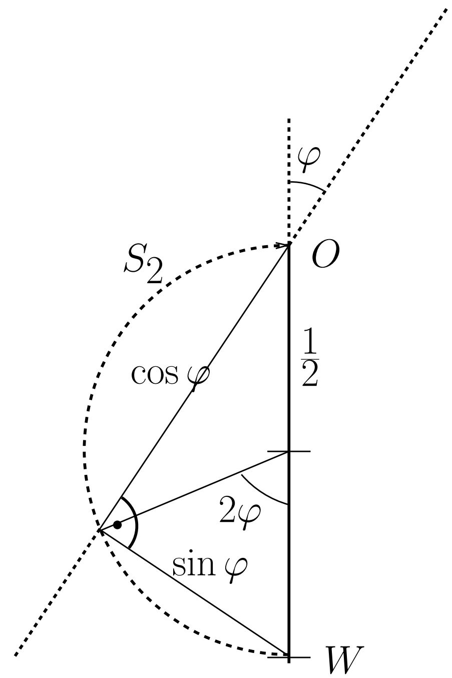
Online exploration arbitrary polgons

- Explore a single vertex
- Looking around a corner
- Simple strat. and analysis: Arc of a circle



Looking around a corner, Theorem

- $H(\varphi) = \varphi$, monotonically increasing: Maximum at $\varphi = \frac{\pi}{2}$
- $G(\varphi) = \frac{\varphi}{\sin\varphi}$, $G'(\varphi) = \frac{\sin\varphi - \varphi\cos\varphi}{\sin^2\varphi}$
- Optimize: $G'(\varphi) > 0$ für $\varphi \in (0, \pi/2]$, Max. at $\varphi = \frac{\pi}{2}$
- Ratio: $\frac{\pi}{2}$ since perimeter is $\pi \times D$



Looking around a corner: Lower bound!

- Special case!
- To the left or right of X!
- In any case! At least $\frac{2}{3}\sqrt{3}$, Blackboard! **Theorem**
- Optimal path: Ratio 1.212. . . **Theorem**