

Online Motion Planning MA-INF 1314

Summersemester 17

Escape Paths/Alternative Measure

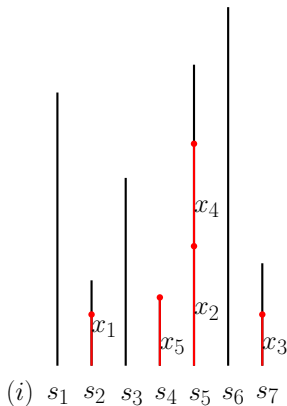
Elmar Langetepe

University of Bonn

Juli 18th, 2017

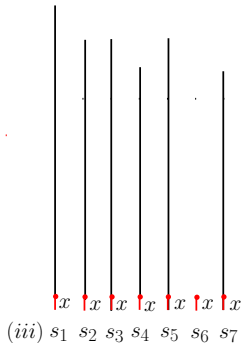
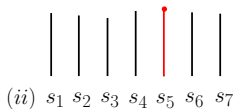
Rep.: Different performance measures

- Set L_m of m line segments s_i of unknown length $|s_i|$
- Dark corridors, escape, digging for oil
- Test corridors successively
- s_{j_1} up to a certain distance x_1 , then s_{j_2} for another distance x_2 and so on



Rep.: Partially informed

- Assume distribution is known!
- $f_1 \geq f_2 \geq \dots \geq f_m$ order of the length given
- Extreme cases! Good strategies!



Rep.: Optimal strategy for this case

Theorem: For a set of sorted distances F_m (i.e. $f_1 \geq f_2 \geq \dots \geq f_m$) we have

$$\max\text{Trav}(F_m) := \min_i i \cdot f_i.$$

Proof:

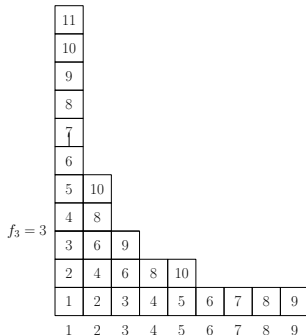
- Arbitrary strategy A
- Less than $\min_i i \cdot f_i$ means less than $j \cdot f_j$ for any j
- Visiting depth $d_1 \geq d_2 \geq \dots \geq d_m$
- Not reached f_1 by d_1 ,
not reached f_2 , since $d_1 + d_2 < 2f_2$ and $d_2 \leq d_1$ and so on
- Not successful!
- $\min_i i \cdot f_i$ always sufficient!

Online Strategy

- F_m with $f_1 \geq f_2 \geq \dots \geq f_m$ not known
- Compete against $\max\text{Trav}(F_m) := \min_i i \cdot f_i$
- Dovetailing strategy: Rounds $c = 1, 2, 3, 4, \dots$
- For any round c from left to right:
Path length of segment i is *extended* up to distance $\lfloor \frac{c}{i} \rfloor$

Rep.: Online Strategy

- Dovetailing strategy: Rounds $c = 1, 2, 3, 4, \dots$
- For any round c from left to right:
Path length of segment i is *extended* up to distance $\lfloor \frac{c}{i} \rfloor$



Theorem: Hyperbolic traversal algorithm solves the multi-segment escape problem for any list F_m with maximum traversal cost bounded by

$$D \cdot (\max\text{Trav}(F_m) \ln(\min(m, \max\text{Trav}(F_m))))$$

for some constant D .

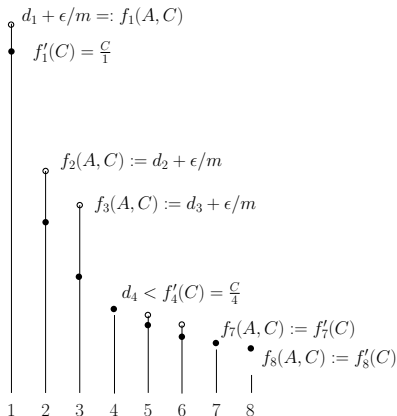
Proof:(W.l.o.g. F_m integers)

- Let $\min_i i \cdot f_i = j \cdot f_j$ for some j
- c with $c = j \cdot f_j$ exists (Round c)
- Overall cost:

$$\sum_{t=1}^m \left\lfloor \frac{c}{t} \right\rfloor \leq \sum_{t=1}^{\min(m,c)} \frac{c}{t} \leq c + \int_1^{\min(m,c)} \frac{c}{t} dt = c(1 + \ln \min(m, c)).$$

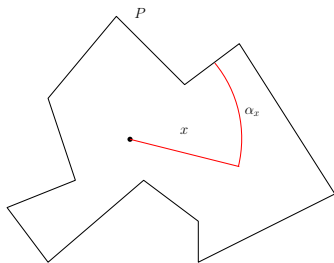
Rep.: Matches Lower bound!

Theorem: For any deterministic online strategy A that solves the multi-segment escape problem we can construct input sequences $F_m(A, C)$ so that A has cost at least $d \cdot C \ln \min(C, m)$ and $\max \text{Trav}(F_m(C, A)) \leq C$ holds for some constant d and arbitrarily large values C .



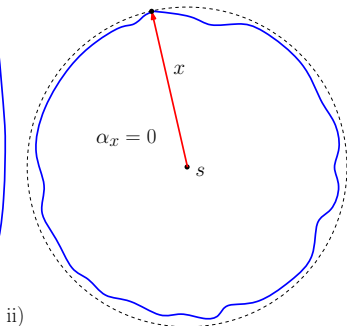
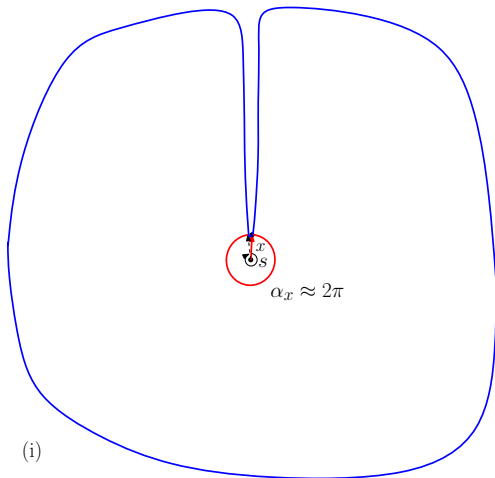
Rep.: Different performance measure: Simple Polygon

- Simple polygon, escape path unknown
- Searching for different cost measure
- Polygonal extension of the list search problem
- Distance to the boundary x (estimation, given)
- Simple circular strategy $x(1 + \alpha_x)$



Rep.: Extreme cases, circular strategy

- Circular escape path: Distribution of the length is known
- Extreme situations: $x_1(1 + 2\pi)$, $x_2(1 + 0)$

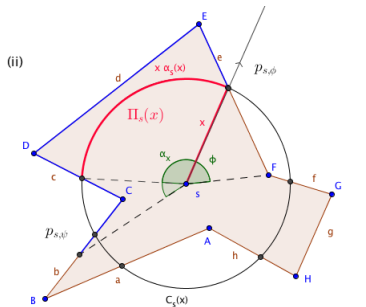
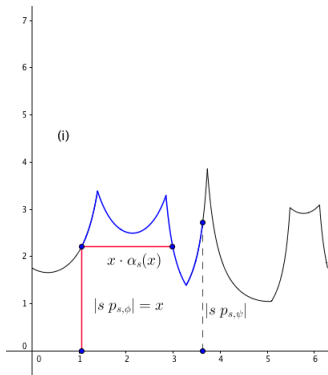


Circular strategy: Star shaped polygon

- Optimal circular escape path for $s \in P$: $\Pi_s(x)$
- For any distance x a worst-case $\alpha_s(x)$
- In total: $\min_x x(1 + \alpha_s(x))$

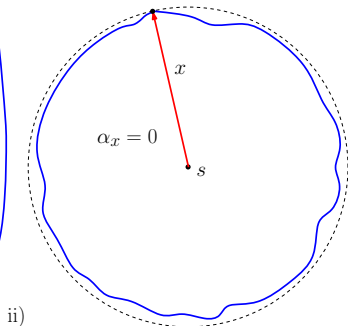
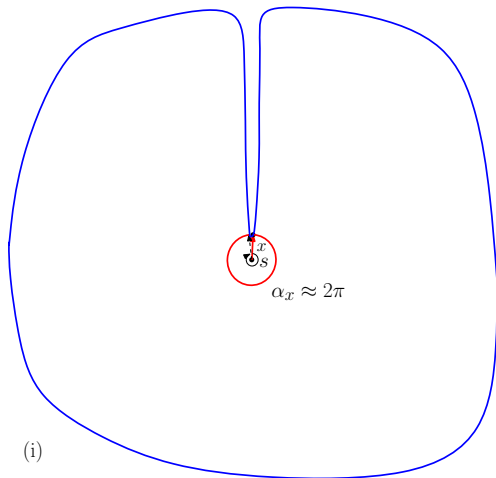
$$\Pi_s := \min_x \Pi_s(x) = \min_x x(1 + \alpha_s(x)) .$$

- Radial dist. function interpretation: Area plus height!



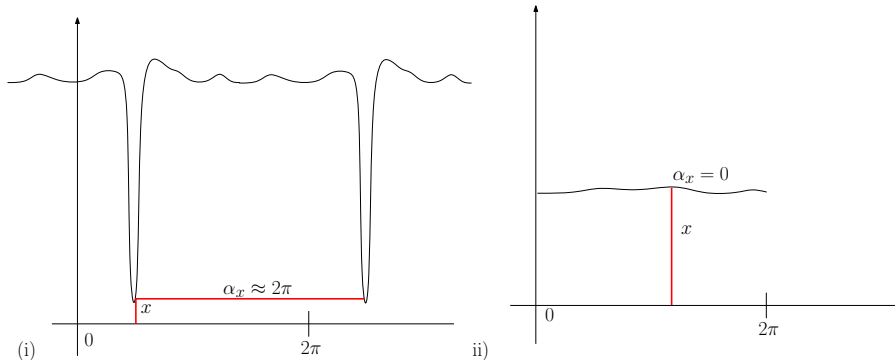
Extreme cases: Radial dist. function

- Circular escape path: Distribution of the length is known
- Extreme situations: $x_1(1 + 2\pi)$, $x_2(1 + 0)$



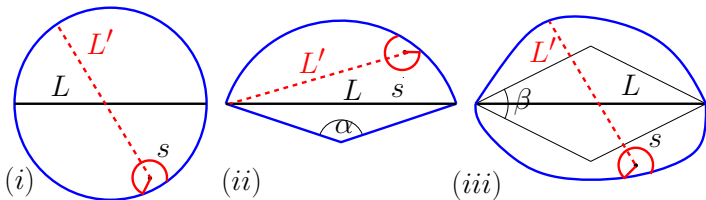
Radial distance function of extreme cases

- Optimal circular escape path
- Hit the boundary by 90 degree wedge
- Area plus height! $\min_x x(1 + \alpha_x)$



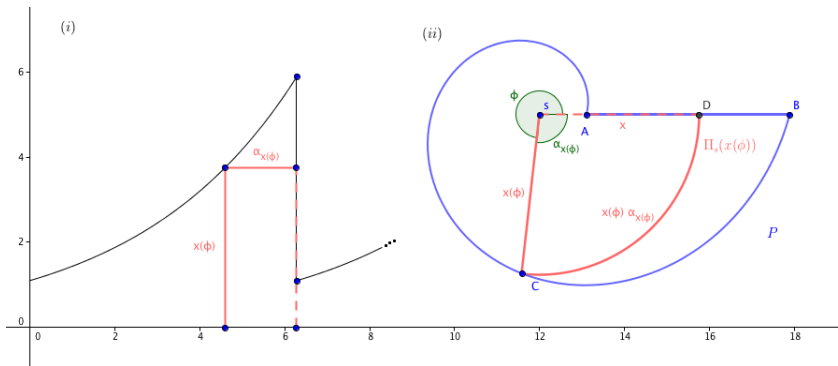
Different justifications

- Simple, computation (polynomial), star-shaped vs. convex
- Natural extension of the discrete certificate (Kirkpatrick)
- Outperforms escape paths for known cases (diameter)



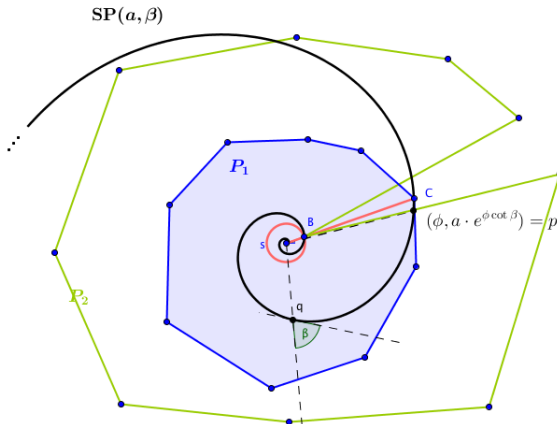
Interesting example

- Distance distribution exactly resembles the polygon
- Analogy to discrete case! Sorting!
- Log. spiral α_x for any x is known:
 $x(\phi) \cdot (1 + \alpha_{x(\phi)})$ with $\alpha_{x(\phi)} = 2\pi - \phi$ and $x(\phi) = A \cdot e^{\phi \cot \beta}$



Online Approximation!

- Inside a polygon P at point s , totally unknown
- Leave the polygon, compare to certificate path for $s \in P$
- Dovetailing strategy (discr. case)! Now spiral strategy (a, β) !



Analysis of a spiral strategy!

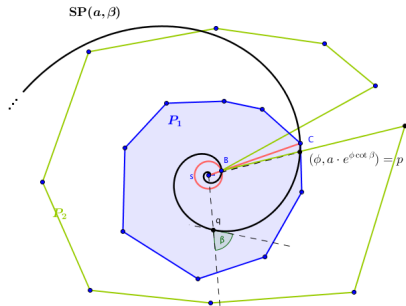
- Assume certificate: $x(1 + \alpha_x)$ for s
- Spiral reach distance $x = a \cdot e^{(\phi - \alpha_x) \cot(\beta)}$ at angle ϕ
- Worst-case success at angle ϕ ! (Increasing for α_x distances!)
- Ratio:

$$f(\gamma, a, \beta) = \frac{\frac{a}{\cos \beta} \cdot e^{\phi \cot \beta}}{a \cdot e^{(\phi - \gamma) \cot \beta} (1 + \gamma)} = \frac{e^{\gamma \cot \beta}}{\cos \beta (1 + \gamma)} \text{ for } \gamma \in [0, 2\pi]$$

- γ represents possible α_x !
- (β, a) represents the spiral strategy!
- Independent from a !
- How to choose β ?

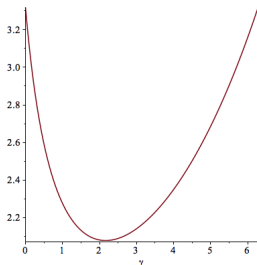
How to choose β ?

- Ratio: $f(\gamma, \beta) = \frac{e^{\gamma \cot \beta}}{\cos \beta (1 + \gamma)}$ for $\gamma \in [0, 2\pi]$
- Balance: Choose β s.th. extreme cases have the same ratio
- $f(0, \beta) = \frac{1}{\cos \beta} = \frac{e^{2\pi \cot \beta}}{\cos \beta (1 + 2\pi)} = f(2\pi, \beta)$
- $\beta = \operatorname{arccot} \left(\frac{\ln(2\pi + 1)}{2\pi} \right) = 1.264714 \dots$



Balance the extreme cases!

- $\beta := \operatorname{arccot} \left(\frac{\ln(2\pi+1)}{2\pi} \right) = 1.264714 \dots$
- Ratio: $f(\gamma, \beta) = \frac{e^{\gamma \cot \beta}}{\cos \beta (1+\gamma)}$ for $\gamma \in [0, 2\pi]$
- $f(0, \beta) = f(2\pi, \beta) = 3.31864 \dots$
and $f(\gamma, \beta) < 3.31864 \dots$ for $\gamma \in (0, 2\pi)$

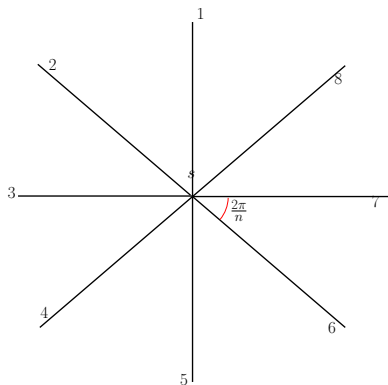


Spiral strategy for $\beta = 1.264714 \dots$

Theorem: There is a spiral strategy for any unknown starting point s in any unknown environment P that approximates the certificate for s and P within a ratio of 3.31864.

Online spiral strategy is almost optimal!

- Lower bounds: Difficult to achieve!
- Arbitrary strategy: Discretization, Reordering, Functionals!

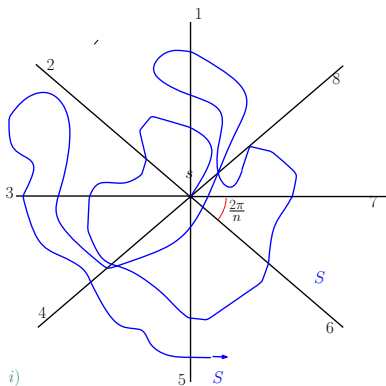


Detour at least $D/2 \geq 3.313126!$

n goes to infinity!

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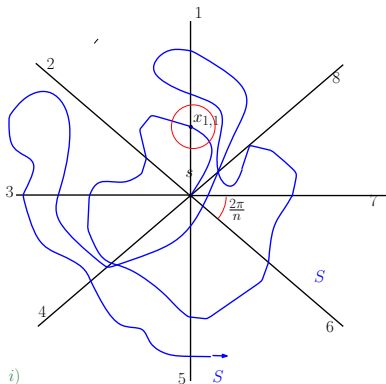


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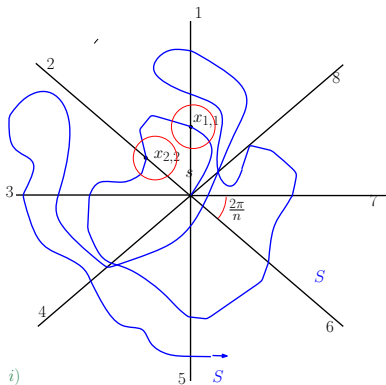


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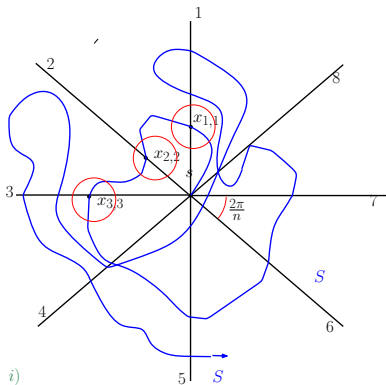


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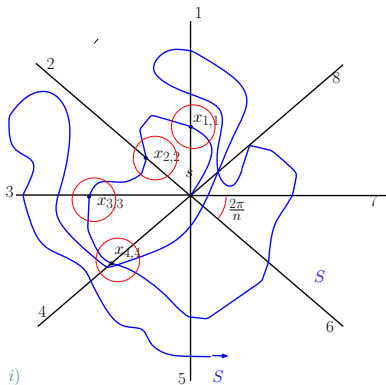


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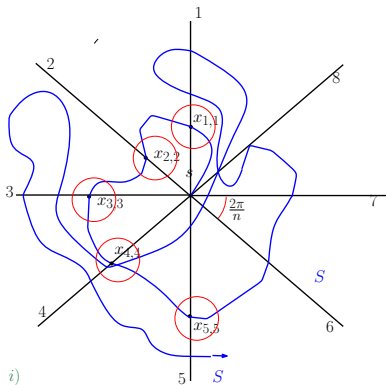


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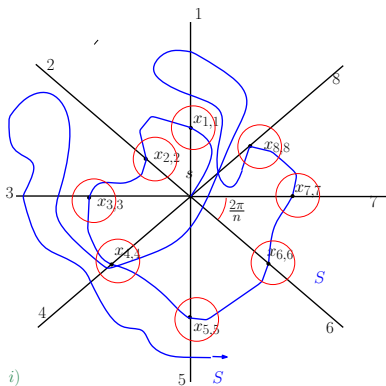
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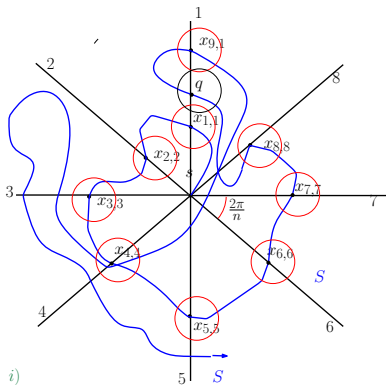


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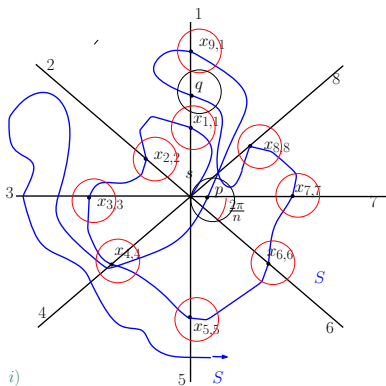


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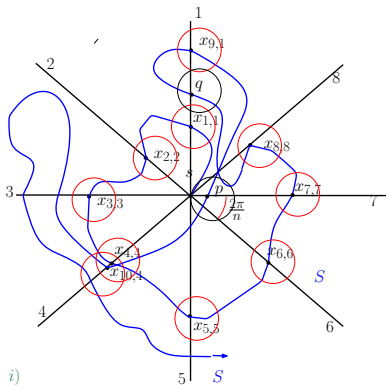


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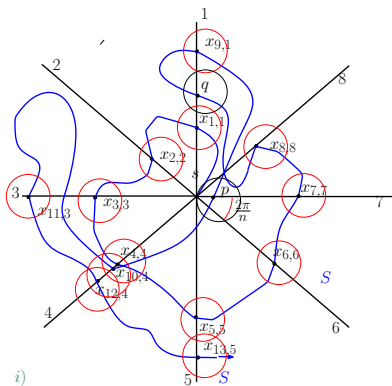


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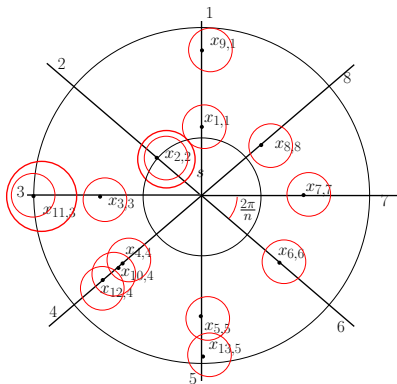


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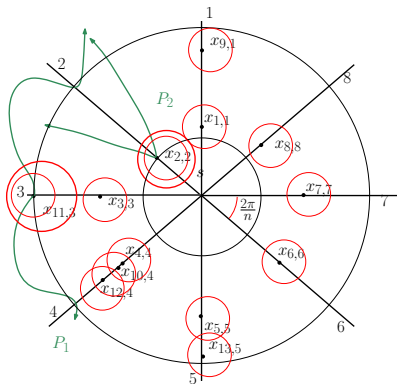
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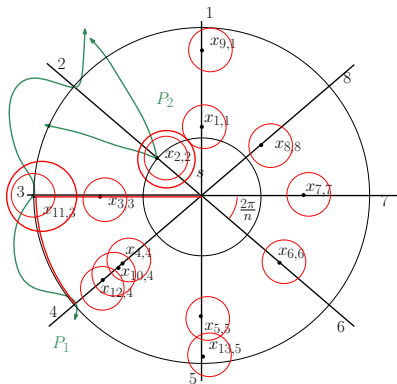


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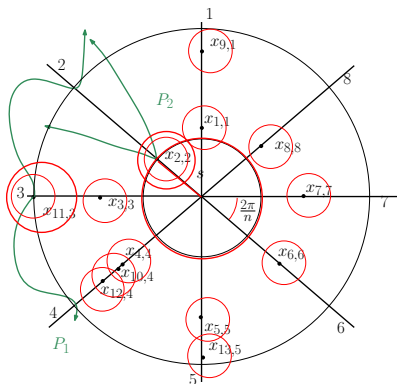


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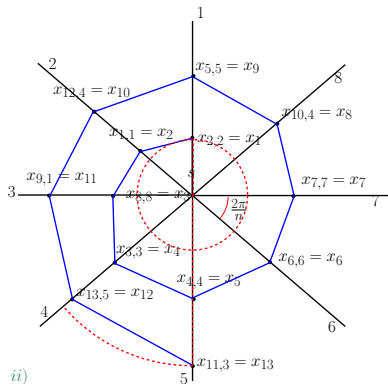


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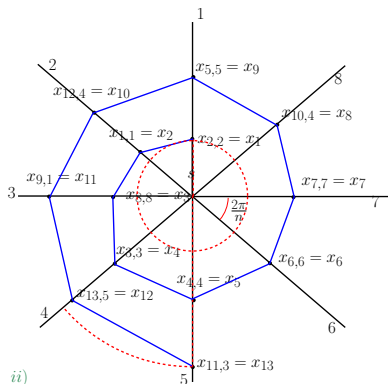


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Lower bound: Minimize

$$\frac{\sum_{i=1}^{k-1} \sqrt{x_i^2 - 2 \cos\left(\frac{2\pi}{n}\right) x_i x_{i+1} + x_{i+1}^2}}{x_k \left(1 + \frac{2\pi}{n}\right)} +$$

$$\frac{\sum_{i=1}^{k-1} \sqrt{x_i^2 - 2 \cos\left(\frac{2\pi}{n}\right) x_i x_{i+1} + x_{i+1}^2}}{x_{k-n+1} (1 + 2\pi)}$$

for all n and k .

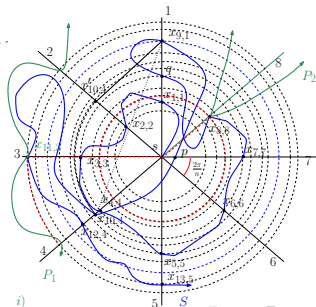
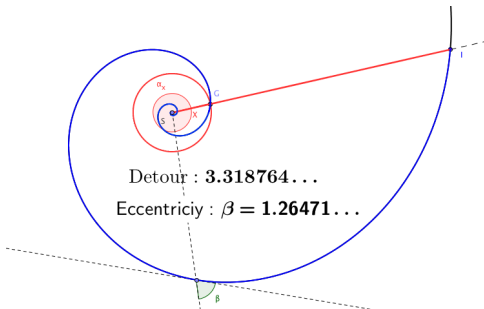
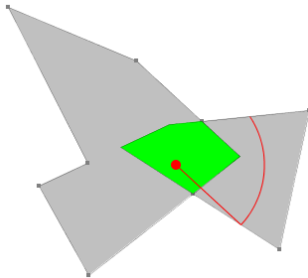
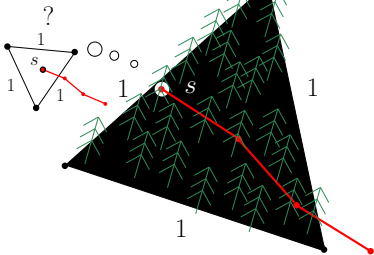
Gives at least $D = 6.62521\dots$ for the sum of Detours!

Detour at least $D/2 \geq 3.313126!$

n goes to infinity!

Summary!

Adversary rotates/translates



Theorem: Escape from an unknown environment against the best certificate can be done within a competitive ratio of $3.318764\dots$ and this is (almost ≈ 0.005) tight.