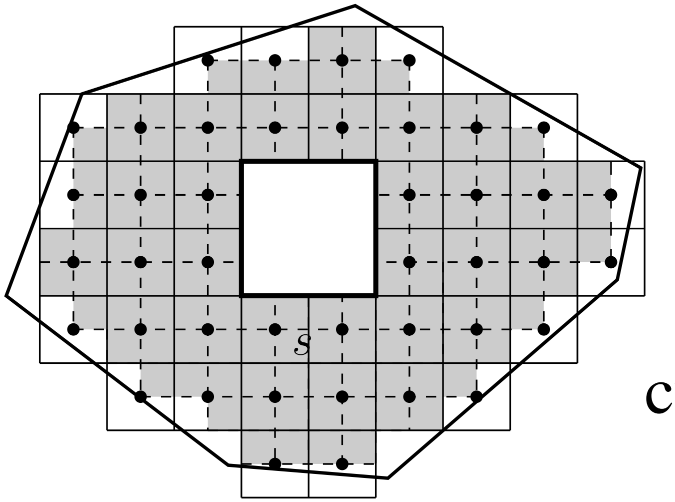
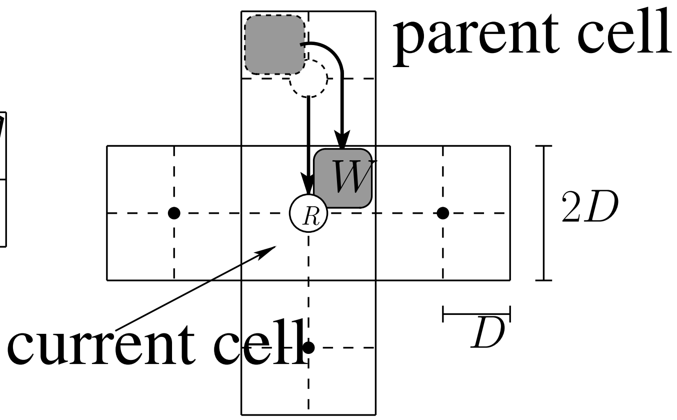
Online Motion Planning MA-INF 1314
General Grids

Elmar Langetepe
University of Bonn

General gridpolygons

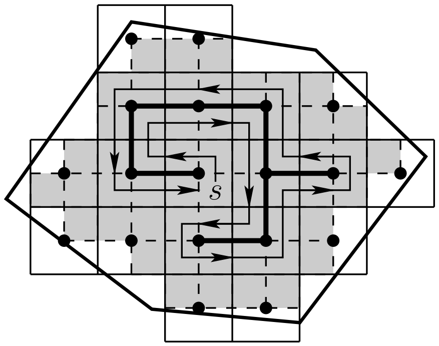
- Change the modell, due to the analysis
- $2D$ cells with center, sub-cells
- *See* adjacent $2D$ Zellen
- Tool W of size D



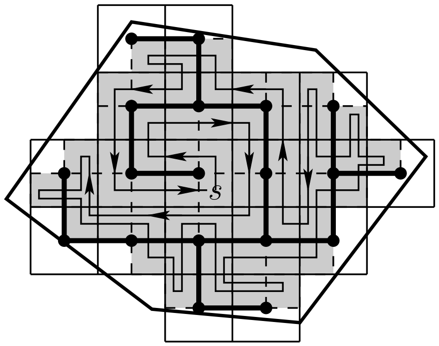


Spanning Trees

- Online with DFS with a Spanning Tree of 2D vertices
- Move the tool along the tree
- Left-Hand-Rule along the tree
- Variants 2D cell totally free for the edge/or not!
- Any cell only once or more than once



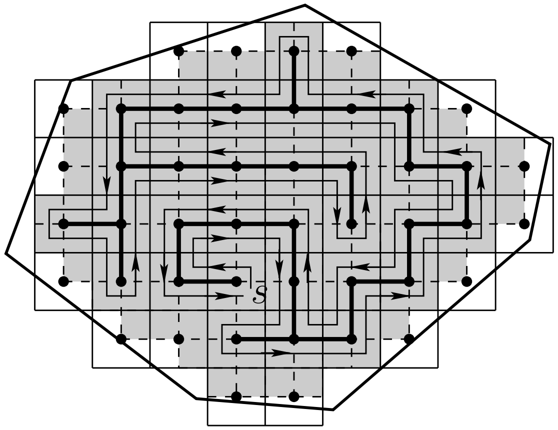
Only full 2D cells



Also partial 2D cells

2D Spiral STC: $\mathbf{2DSPSTC}(\boldsymbol{parent}, \boldsymbol{current})$

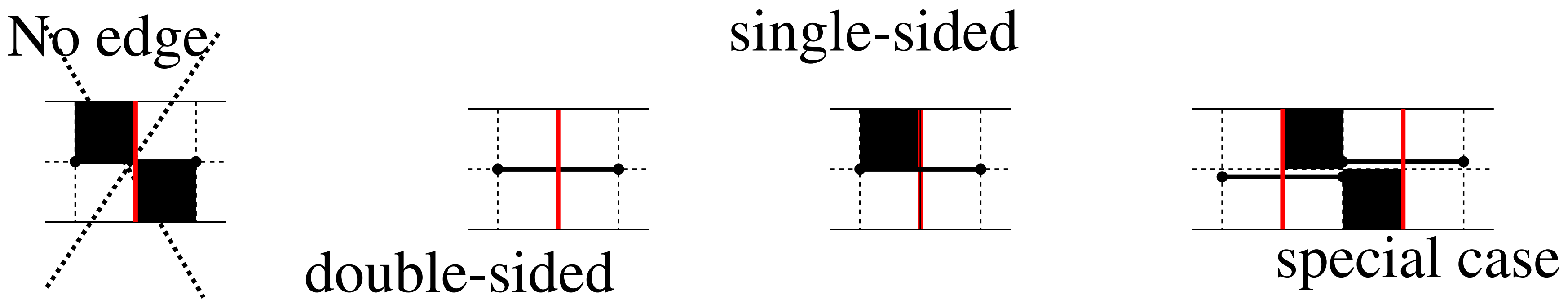
```
Mark current as explored
while current has unexplored neighbour do
        • Search from parent in ccw order
            neighbour free non-explored/free
        • Span.Tree edge current zu free.
        • Move tool L-H-R along
            Span.Tree edge to
            first sub-cell of free
        • 2DSPSTC( current, free )
end while
if  current ≠ $s$  then
        • From current by L-H-R along
            Span.Tree to subcell of parent
end if
```



Only full 2D cells

Partially blocked 2D cells

- Spanning Tree, edge is free/not the full cell
- Reachable D sub-cells?
- Different types
- **Definition:** double-sided edge, one-sided edge



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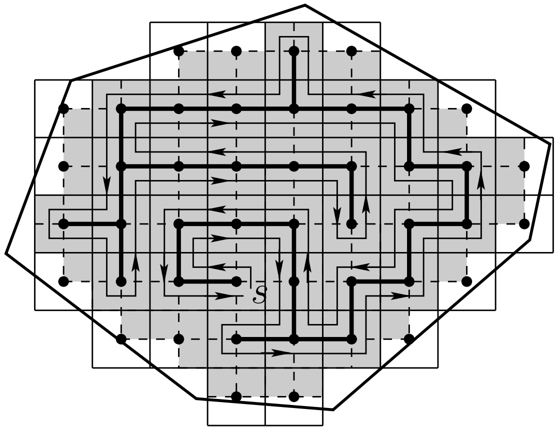
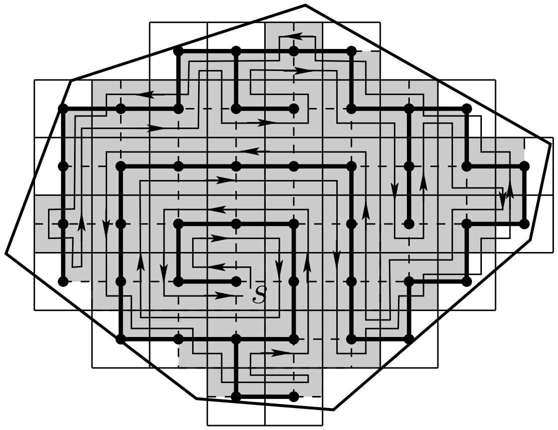


Only full 2D cells

Spiral STC: $\mathrm{SPSTC}(\boldsymbol{parent}, \boldsymbol{current})$

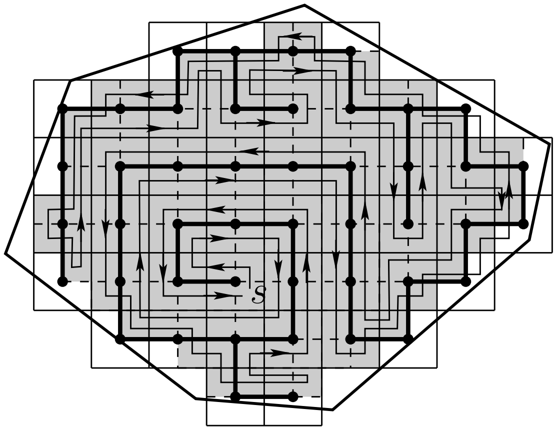
- Search from *parent* in ccw order neighbour *free*/non-explored, s.th. spanning tree edge can be build (might be single-sided)
- Search from *parent* in ccw order neighbour *free* non-explored/free



Vertex- can-be-reached

Spiral STC: $\mathrm{SPSTC}(\boldsymbol{parent}, \boldsymbol{current})$

- Move tool along the spanning tree edge to the first reachable sub-cell of *free*. Left-Hand-Rule for double-sided edges.
  Avoid obstacles of single-sided edges.
  Tool might change to the left of the spanning tree edge.
- Move tool L-H-R along Span.Tree edge to first sub-cell of *free*



Vertex- can-be-reached

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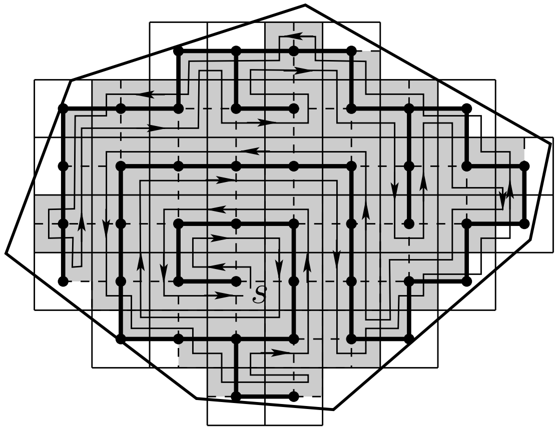
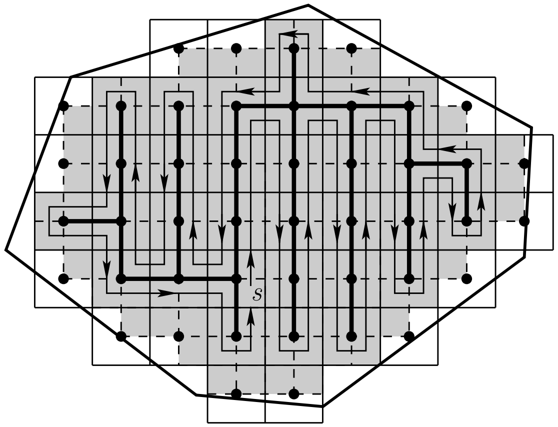
- Move tool along the spanning tree edge to the first reachable sub-cell of *free*. Left-Hand-Rule for double-sided edges. Avoid obstacles of single-sided edges. Tool might change to the left of the spanning tree edge.
- From *current* by L-H-R along Span.Tree to subcell of *parent*

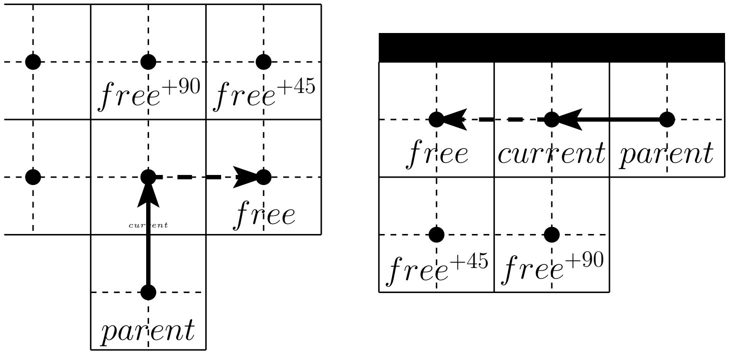


Vertex- can-be-reached

Less rotations for the tool

- Avoid spiral-like paths
- Move in columns
- Scan also diagonally adjacent 2D cells
- ScanSTC 2D Algorithm
- Also for the Backtracking step

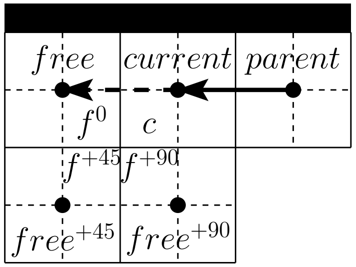


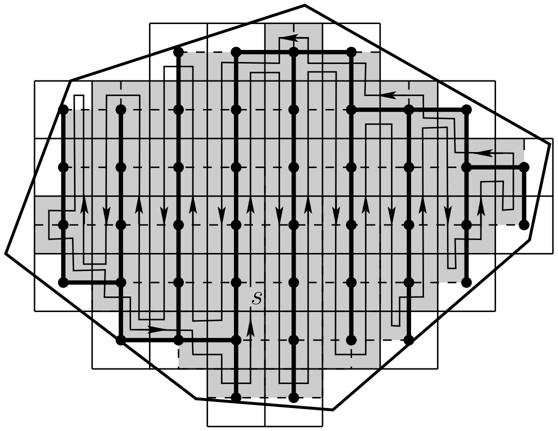


(i) (ii)

Less rotations for the tool

- Avoid spiral-like paths
- Move in columns
- Also for the general case/path should exist
- Scan also diagonally adjacent 2D cells
- ScanSTC Algorithm
- Also for the Backtracking step





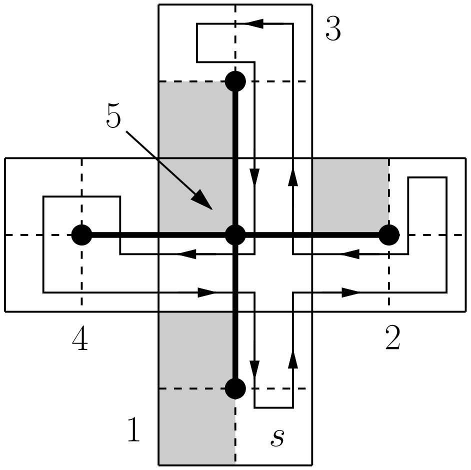
Analysis! Theorem

- General Spiral STC
- Number of steps for the tool
- As given by SmartDFS, C plus overhead
- D sub-cells, at the *boundary* K in total

P gridpolygon, C reachable sub-cells. K reachable sub-cells that are diagonally adjacent to a blocked sub-cell. P is explored by Spiral-STC or Scan-STC. Requires $O(C)$ space and $O(C)$ time. The number of steps for the tool is restricted to $S \leq C + K$.

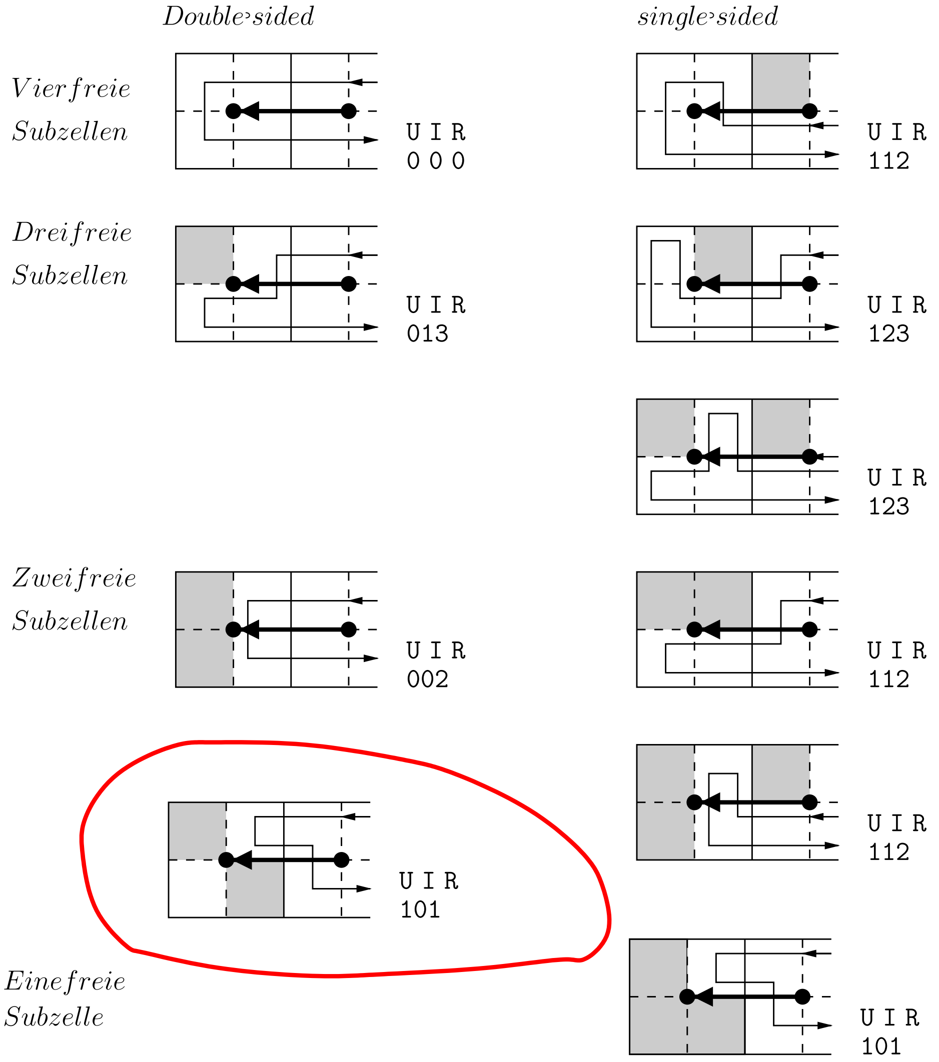
Number of steps! Example!

- Locally, count boundary sub-cells
- Local analysis, multiple visits, charge boundary sub-cells
- 2D Inner-cell/ Intra-cell
- Systematically: Boundary sub-cells charged by *Inner* plus *Intra*



Cell	Intra	Inner	Full	Bd.-cells
1	0	1	1	2
2	1	2	3	3
3	1	2	3	3
4	1	1	2	2
5	1	2	3	3

Number of steps! Theorem Systematically!



Double›sided *single›sided*

Vierfreie Subzellen

U I R 0 0 0

U I R 112

Dreifreie Subzellen

U I R 013

U I R 123

U I R 123

Zweifreie Subzellen

U I R 002

U I R 112

U I R 101

U I R 112

Einefreie Subzelle

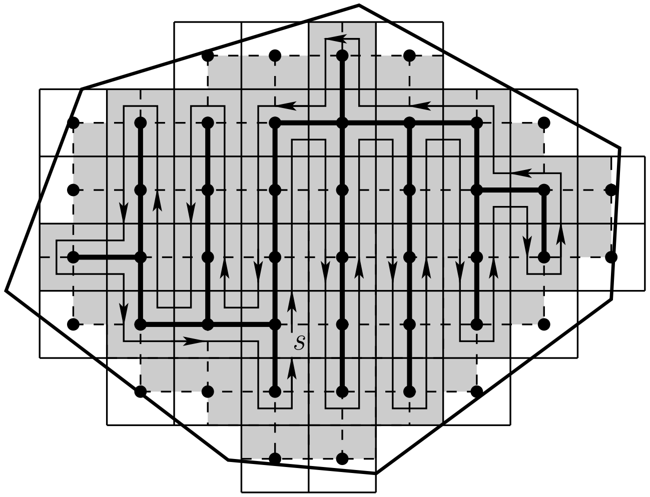
U I R 101

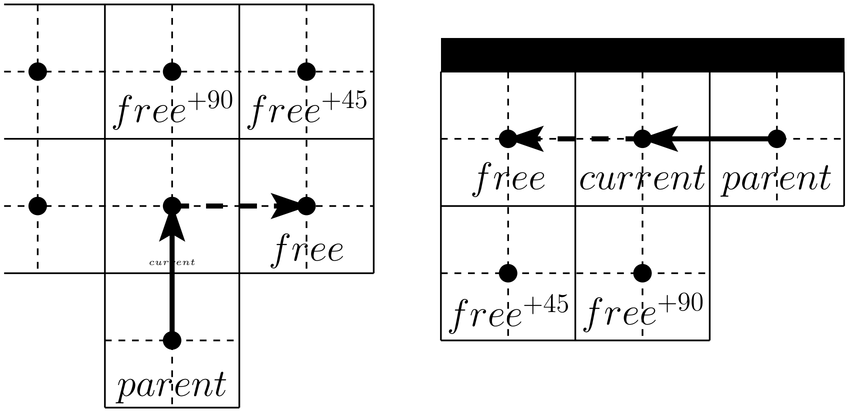
Running time and space required Theorem

- $C + K$ steps but K is bounded by C
- Local decisions: $O(1)$
- Running time and space $O(C)$

Analysis of 2D-ScanSTC

- Give a Scan-Preference: I.e. Vertically
- Decide only locally (more information)
- How many *bad* horizontal edges?
- Optimal numberl H_{opt}
- Compare with 2D-ScanSTC: Say H_{STC}

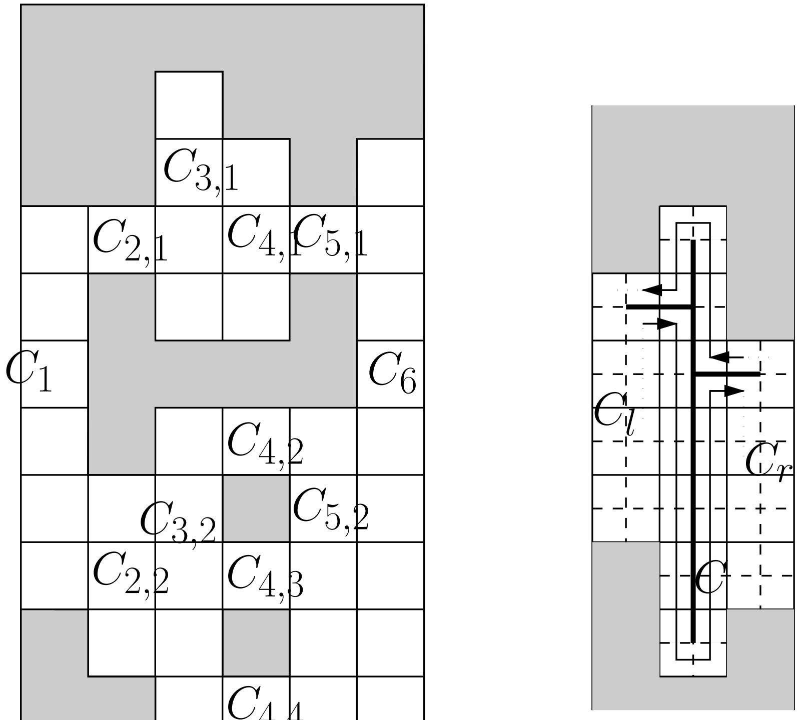




(i) (ii)

Analysis of 2D-ScanSTC

- Columns connectivity
- From Left to Right X nach Y
- Sum up the Differences: Overall Z
- Connectivity changes



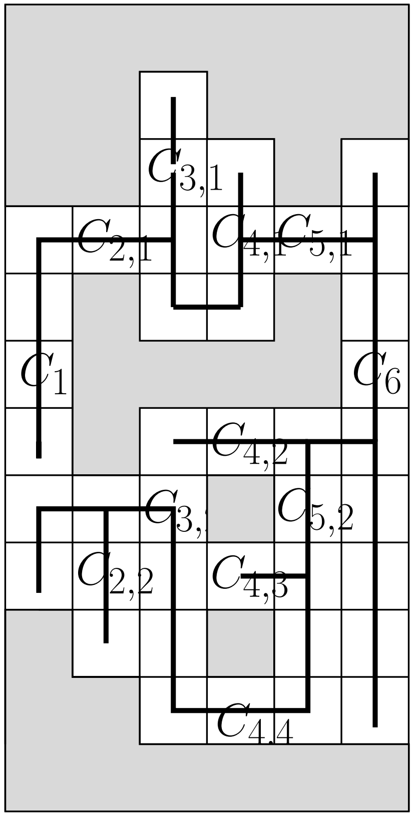
(i) (ii)

Proof Sketch

- H_{Opt} optimal number of horizontal edges in the spanning tree. Z number of connectivity changes of P. $2D$-Scan-STC requires

$$H_{STC} \leq H_{Opt} + Z + 1$$

horizontal edges in its spanning tree.



$C_{3,1}$
$C_{2,1}$
$C_{4,1}$
$C_{5,1}$
C_1
C_6
$C_{4,2}$
$C_{3,}$
$C_{5,2}$
$C_{2,2}$
$C_{4,3}$
$C_{4,4}$