Online Motion Planning MA-INF 1314
Restricted Graphexploration

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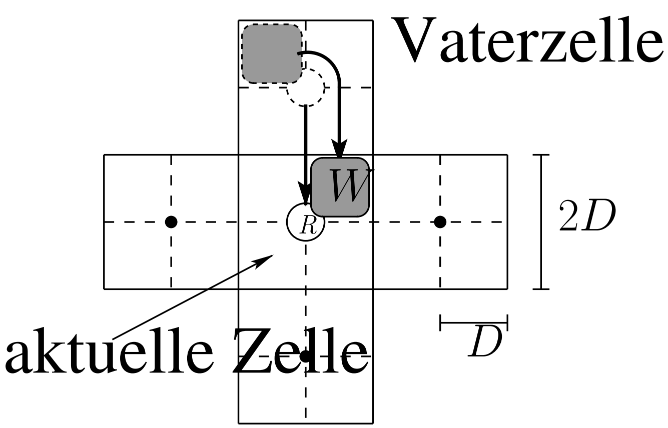
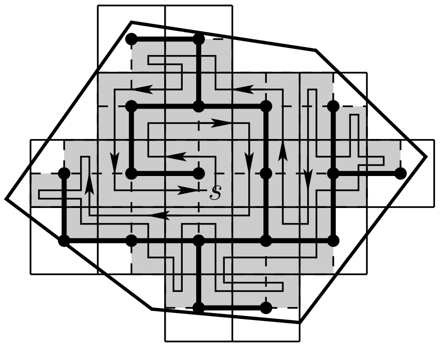
Repetition!

- Modell 2D-cells, Spanning-Tree online construction
- SpiralSTC/ScanSTC: Detours along Spanning-Tree edge
- SpiralSTC equivalent to sub-cell-Modell!!
- Algorithmic formulation, recursively defined
- Strategy-Analysis: Locally!

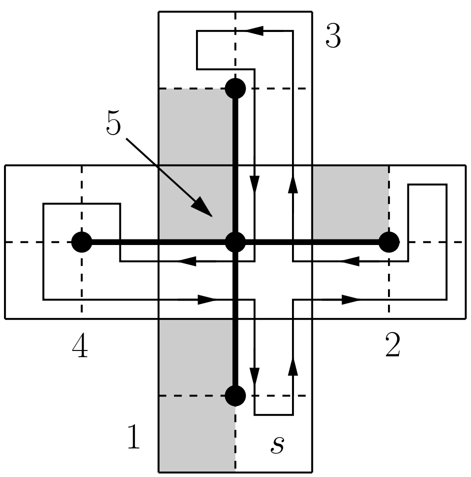




Belegt aber begehbare 2D Zellen

Repetition: Local analysis

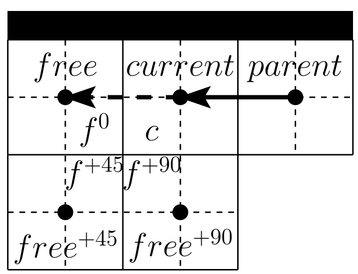
- Count the boundary cells
- Local analysis, multiple visits of cells, charge 2D cell
- Inner-cell (Responsibility)), Intra-cell
- Systematically: Boundary D-cells $\geq$ inner+intra
- Theorem: $C + K$ (tight!)

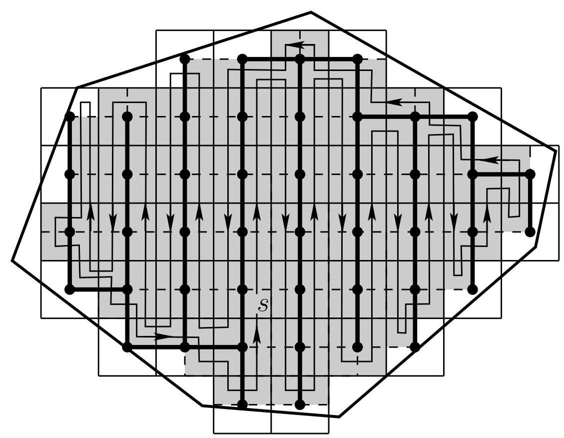


Cell	Intra	Inner	Full	Bd.-cells
1	0	1	1	2
2	1	2	3	3
3	1	2	3	3
4	1	1	2	2
5	1	2	3	3

Repetition: Less rotations for the tool

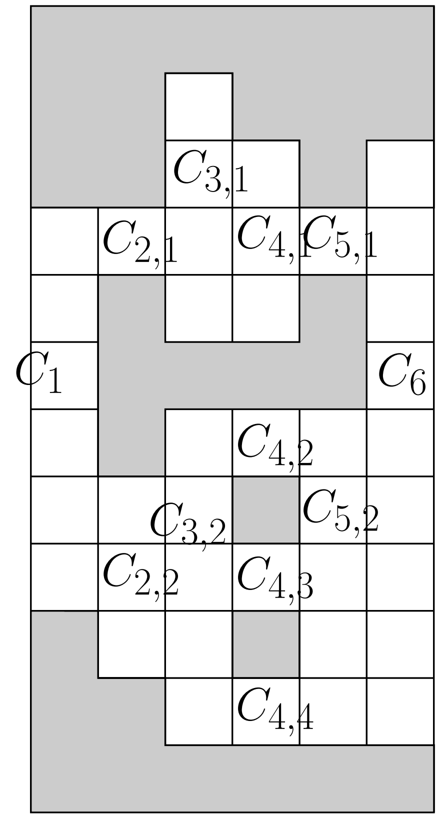
- Avoid spiral-like paths
- Move in columns
- Also for the general case/path should exist
- Scan also diagonally adjacent 2D cells
- ScanSTC Algorithm
- Also for the Backtracking step



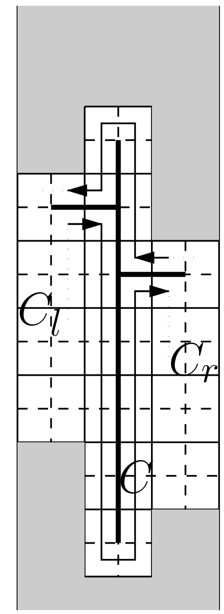


Analysis of 2D-ScanSTC

- Columns connectivity
- From Left to Right X nach Y
- Sum up the Differences: Overall Z
- Connectivity changes



(i)



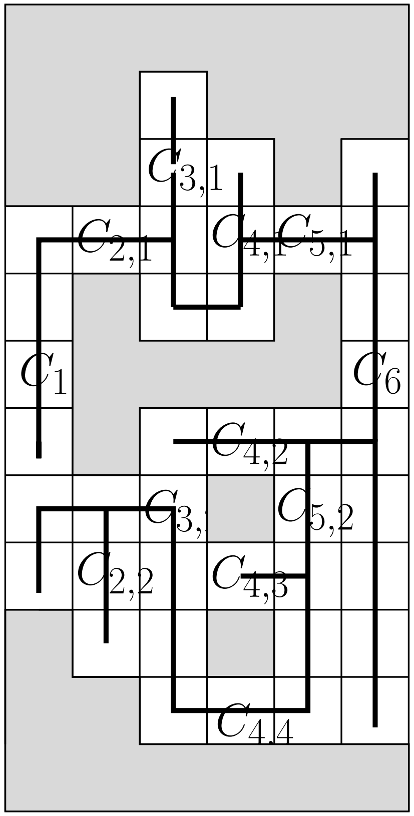
(ii)

Proof Sketch

- H_{Opt} optimal number of horizontal edges in the spanning tree. Z number of connectivity changes of P. $2D$-Scan-STC requires

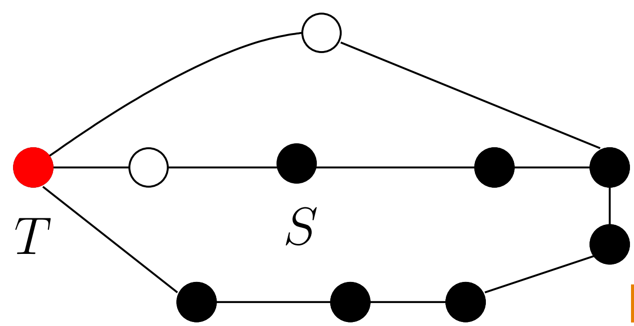
$$H_{STC} \leq H_{Opt} + Z + 1$$

horizontal edges in its spanning tree.



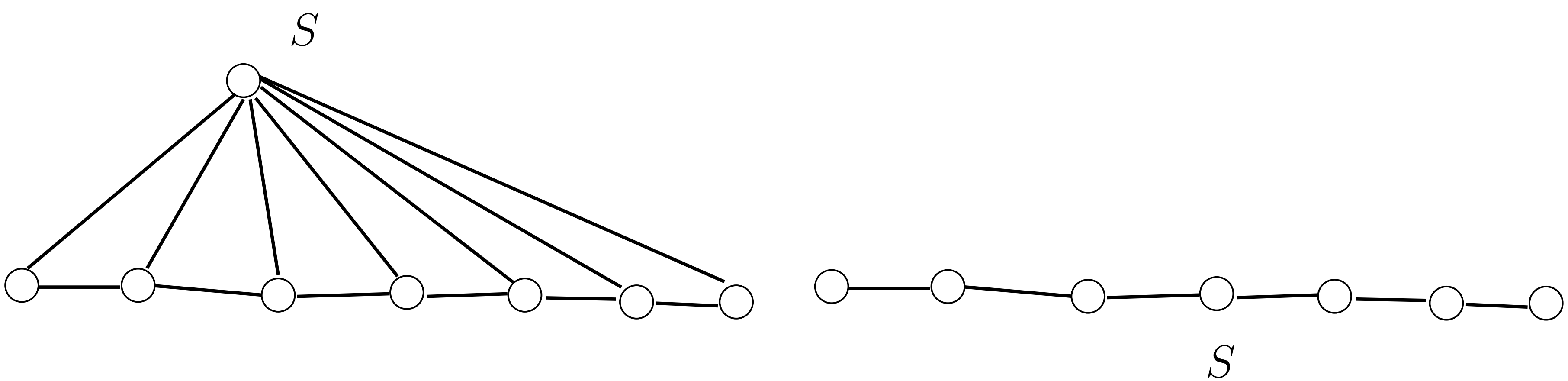
Online graphexploration!

- Graph G: Visit all edges and vertices
- DFS 2 competitive, optimal
- Searching $\Rightarrow$ Not too much into the depth
- Restricted exploration, tether/accum. (applications)



Restricted online graphexploration

- Tether of length k
- Graph G: Depth k, longest shortest path to start
- Pure DFS: $k = 1$ but tether length n is required
- BFS: $k \approx n/2$ but $\Omega(n^2)$ visits for n edges



Modell: Restricted (online) graphenexploration

1. Tethered agent $l = (1+\alpha)r$ (cable).
2. Agent returns to start after $2(1+\alpha)r$ steps (recharge accumulator)
3. Large graph, explore up to depth d, flexible d

- All vertices r steps away, depth r (radius)
- All edges length 1 (weights, exercise)
- Small look-ahead α necessary
- First variant, reduction for the others (Lemma/Exercise)

Restricted graphexploration: Simulation

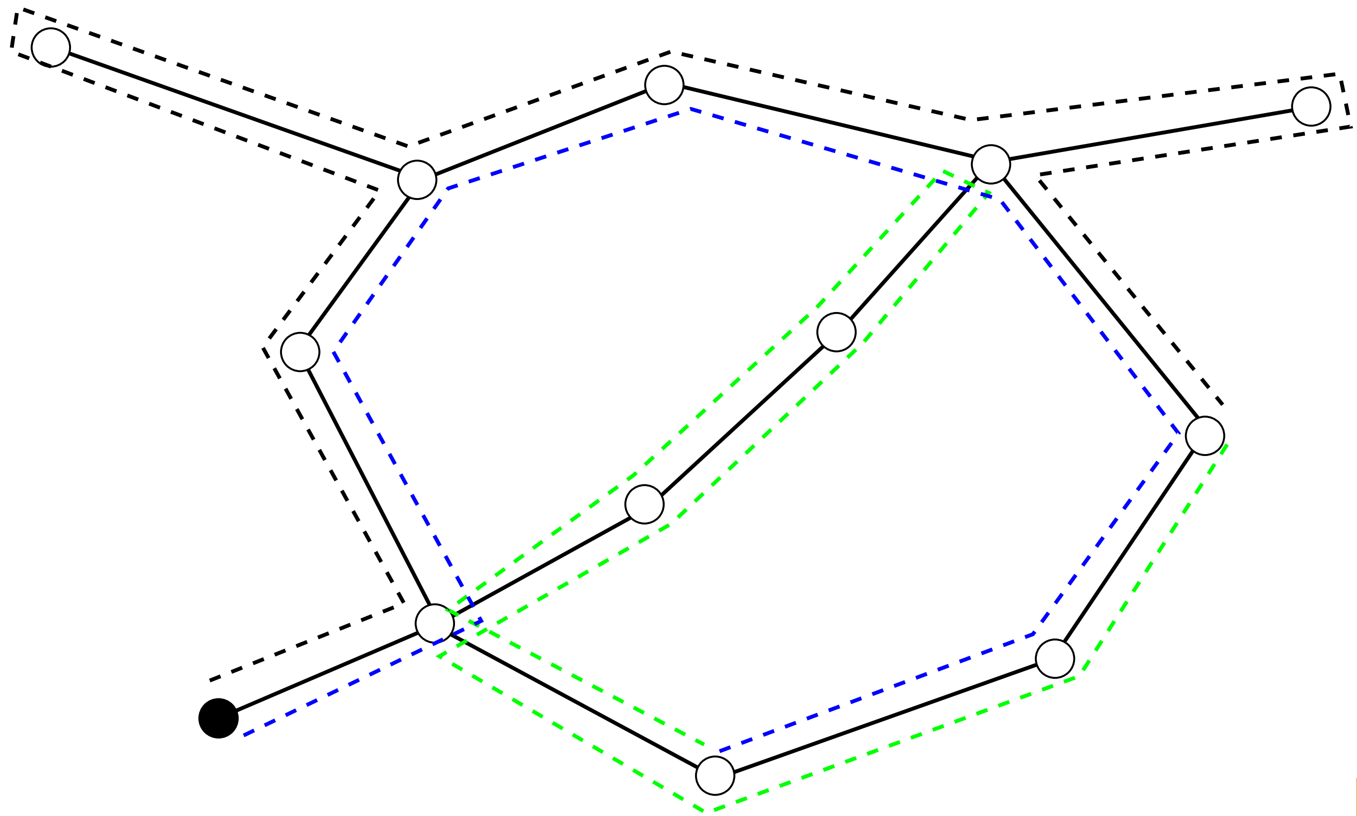
Lemma For any $\beta > \alpha$ a solution for the accumulation-variant with accumulator size $2(1+\beta)r$ can be attained from the solution of the tethered-variant with tether length $l = (1+\alpha)r$. The cost decrease by a factor of $\frac{1+\beta}{\beta-\alpha}$.

Proof: Blackboard!!!

Offline Algorithmus: Accumulator-variant

- Offline: Graph is fully known
- Assume: $4r$ Accumulator
- Complexity, (NP-hard ?) unknown! Approximation $O(|E|)$!
- Algorithm: DFS $2|E|$ steps
- Cut into pieces of length $2r$, subpaths
- Starting segment in distance r
- Visit from start, explore subpath, move back!

Example offline!



$$\left\lceil \frac{2|E|}{2r} \right\rceil \times 2r + 2|E| \leq 6|E|$$ Example: $r = 5$

Offline Algorithm: Accumulator-variant

Lemma A simple Accumulator-Offline Algorithm visits at most $6|E|$ edges.

- Reach any subpath-start with step-length $2r$
- Explore all subpath: $2|E|$
- $\left\lceil \frac{2|E|}{2r} \right\rceil$ subpaths in total
- Reaching by $\left\lceil \frac{|E|}{r} \right\rceil 2r$ steps
- $\left\lceil \frac{|E|}{r} \right\rceil 2r \leq \left(\frac{|E|}{r} + 1 \right) 2r \leq 2|E| + 2r$
- $4|E| + 2r \leq 6|E|$

Online: Tethered graphexploration

- Tether variant (cable), reductions for others (Lemma/Exercise)
- First idea, DFS (edges) until tether is fully used, then backtracking
- bDFS, bounded DFS
- Nice try, is not enough!

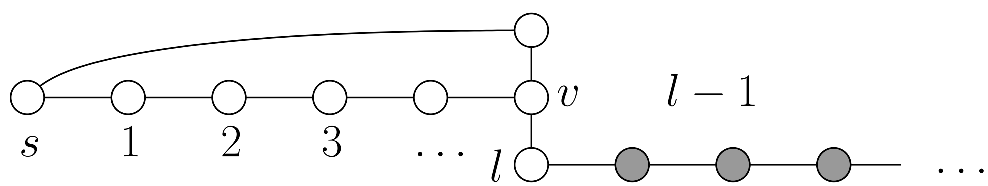
Method: Bounded DFS

bDFS(v, l):

- **if** $(l = 0) \vee$ (all outgoing edges are explored) **then**
  RETURN
  end if
  for all non-explored edge $(v, w) \in E$ **do**
  Move from *v* to *w* by (v, w).
  Mark (v, w) as *explored*
  bDFS(*w*, $l - 1$).
  Move back from *w* to *v* by (v, w).
  end for

Bounded DFS

- Example unit-length edge
- Problem: Not all edges will be reached
- Edge to v is marked, End!
- Only bDFS is not enough



CFS Algorithm: Mark the vertices

non-explored vertices, never visited.

incomplete visited vertices, but there are non-explored edges starting at v.

explored vertices, all incident edges have been explored.

CFS Algorithm

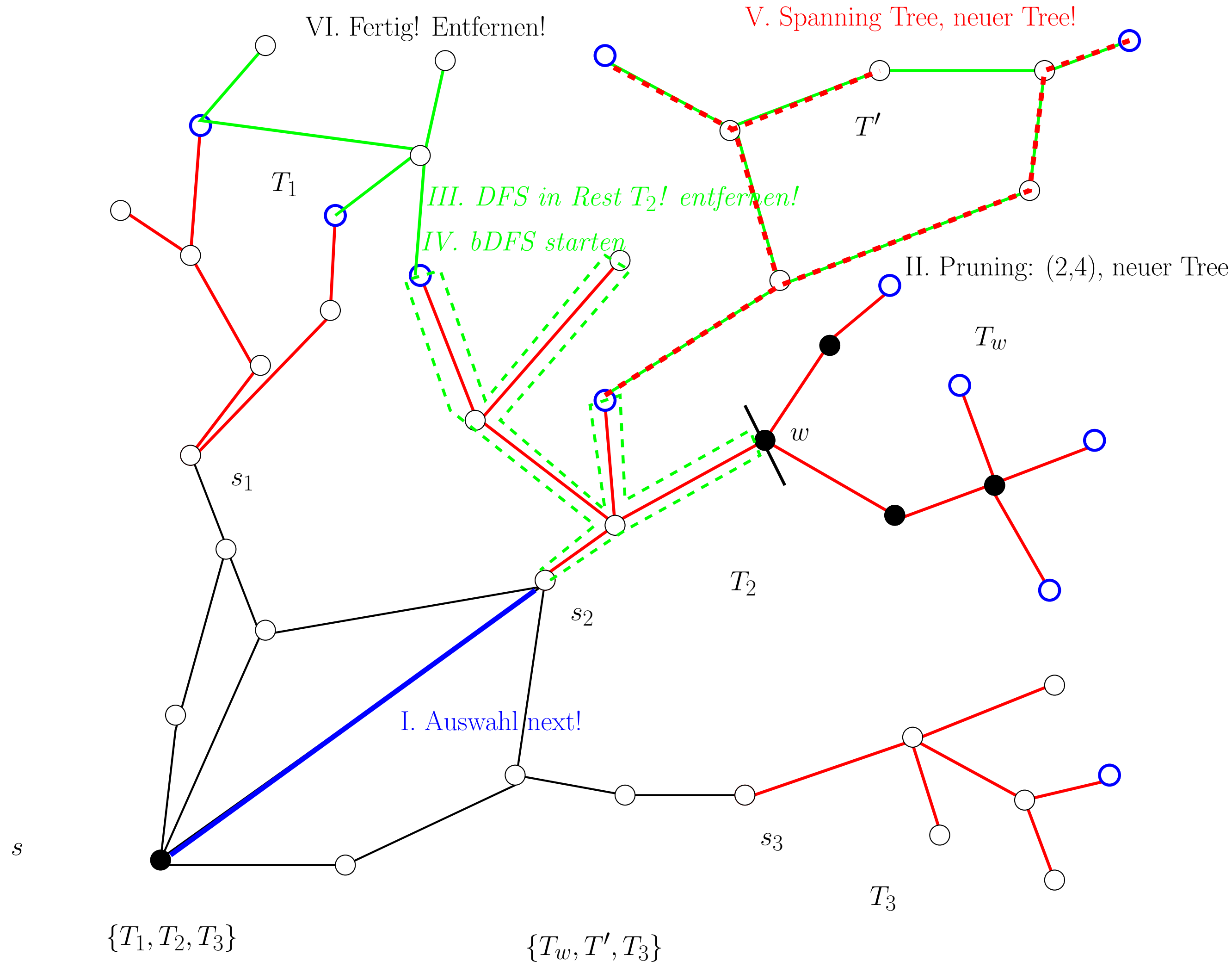
- Start bDFS at different sources
- Set of (edge) disjoint **trees** $\mathcal{T} = \{ T_1, T_2, \ldots, T_k \}$
- Root vertices $s_1, s_2, \ldots, s_k$
- Choose T_i with s_i closest to s, move to s_i
- Pruning of T_i: Build T_{w_j} with root w_j if:

  1. $d_{T_i}(s_i, w_j) \geq \mathit{minDist} = \frac{\alpha r}{4}$
  2. $\mathit{Depth}(T_{w_j}) \geq \mathit{minDepth} - \mathit{minDist} = \frac{\alpha r}{4}$

- Add all T_{w_j} to $\mathcal{T}$! Remove T_i from $\mathcal{T}$
- Explore T_i without T_{w_j} from s_i by DFS and
- start bDFS at the incomplete vertices

- Graph G' of new vertices and edges
- Build a spanning tree T' of G
- Choose root s' with minimal distance to s
- Add all these trees to $\mathcal{T}$
- Special case: Trees in $\mathcal{T}$ gets fully explored
- Trees in $\mathcal{T}$ with common egdes are joined
- Merging: Build spanning tree with new root

CFS Algorithm, Example



CFS Algorithm

CFS(s, r, α)

- $\mathcal{T} := \{\{s\}\}$.

  repeat

  T_i:= tree in G^* closest to s.
  s_i:= root of T_i (closest vertex to s).
  $(T_i, \mathcal{T}_i) :=$ prune(T_i, s_i, $\frac{\alpha r}{4}$, $\frac{\alpha r}{2}$).
  $\mathcal{T} := \mathcal{T} \setminus \{T_i\} \cup \mathcal{T}_i$.
  explore($\mathcal{T}$, T_i, s_i, $(1+\alpha)r$).
  Remove all fully explored trees from $\mathcal{T}$.
  Merge all trees in $\mathcal{T}$ with common vertices.
  Calculate spanning tree/root for merged trees.

  until $\mathcal{T} = \emptyset$

CFS Algorithmus: Pruning!

prune($\boldsymbol{T}$**,** $\boldsymbol{v}$**,** $\boldsymbol{minDist}$**,** $\boldsymbol{minDepth}$ **)**

- $v :=$ Root of T.
  for all $w \in T$ such that $d_T(v, w) = minDist$ **do**
  $\quad$ $T_w :=$ subtree of T with root w.
  $\quad$ **if** max. distance from v and vertex in $T_w > minDepth$ **then**
  $\quad\quad$ // Cut-Off T_w from T:
  $\quad\quad$ $T := T \setminus T_w$.
  $\quad\quad$ $\mathcal{T}_i := \mathcal{T}_i \cup \{T_w\}$.
  $\quad$ **end if**
  end for
  RETURN $(T, \mathcal{T}_i)$

CFS Algorithmus: Explore!

explore($\mathcal{T}$**,** $\boldsymbol{T}$**,** s_i**,** l **)**

- Move from s to s_i along shortest (known) path.
  Explore T by DFS. If incomplete vertex v is visited:
    $l' :=$ remaining tether length.
    bDFS(v, l').
    $E' :=$ newly explore edges.
    $V' :=$ vertices from in E' (plus v).
    Build spanning tree T' of $G' = (V', E')$.
    $\mathcal{T} := \mathcal{T} \cup \{T\}$.
  Move back from s_i to s.

CFS Algorithmus: Example!!

- $G^* = (V^*, E^*)$ Graph of the explored edges and and vertices (successively extended)
- Set $\mathcal{T}$
- Pruning
- Explore (DFS/bDFS)