# Online Motion Planning MA-INF 1314
# Pledge with sensor errors

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# Graph-exploration

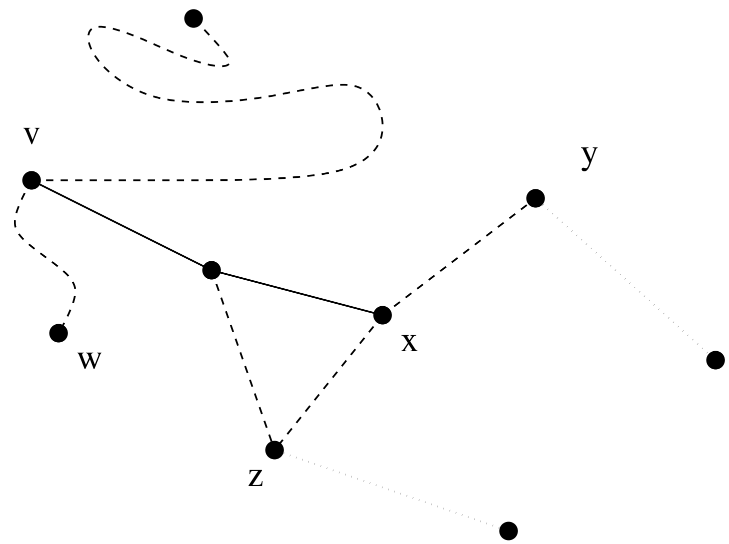
- Labyrinths, grid-graphs, gridpolygons, general graphs
- Graph-exploration: DFS and LB of 2
- Gridpolygons: Simple/general
- SmartDFS $\frac{4}{3}$, LB $\frac{7}{6}$
- STC Alg. $|C| + |B|$
- Tether/Accumulator/Depth variants: $\Theta(|E| + |V|/\alpha)$
- Marker Algorithm
- Online TSP for planar graphs!

# Rep.: Shortcut algorithm

- Online DFS, boundary edge $(x, y)$ *blocks* boundary edge $(v, w)$

  1. $|xy| < |vw|$ and
  2. $d_G(v, x) + |xy| < (1 + \delta)|vw|$

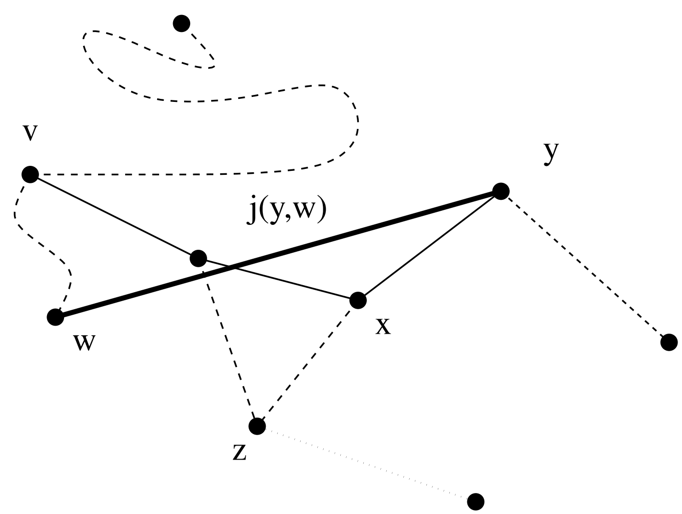
- Later: You eventually move from $y$ over $x$ to $v$!



# Rep.: Shortcut algorithm

- *Shortcut* non-blocked boundary edge $(x, y)$, charge edge $(x, y)$
- Blocked edge $(v, w)$ becomes unblocked by visit of $y$
- Build jump edge $j(y, w)$ of length $d_G(v, w) < (2 + \delta)|vw|$, if visited, charge $(v, w)$

1. $|xy| < |vw|$ and
2. $d_G(v, x) + |xy| < (1 + \delta)|vw|$



# Rep: Pseudocode: Shortcut($x$, $y$, $G$ )

Traveling from $x$, we have visited $y$ for the first time

**for all** Boundary edges $(v, w)$ **do**

  **if** Visiting $y$ caused $Block((v, w))$ to become empty **then**

    Build jump edge $j(y, w)$, append jump edge

      on $Incident(y)$ and $Incident(w)$

  **end if**

**end for**

**for all** Edges $(y, z) \in Incident(y)$ **do**

  **if** $z$ is boundary vertex and $(y, z)$ shortcut **then**

    Traverse the egde $(y, z)$; Shortcut($y$, $z$, $G$ )

  **else if** $z$ is boundary vertex and $(y, z)$ is jump edge **then**

    Move to $z$ along the shortest known path; Shortcut($y$, $z$, $G$ )

  **end if**

**end for**

Return to $x$ along the shortest known path

# Rep: Shortcut algorithm/Analysis

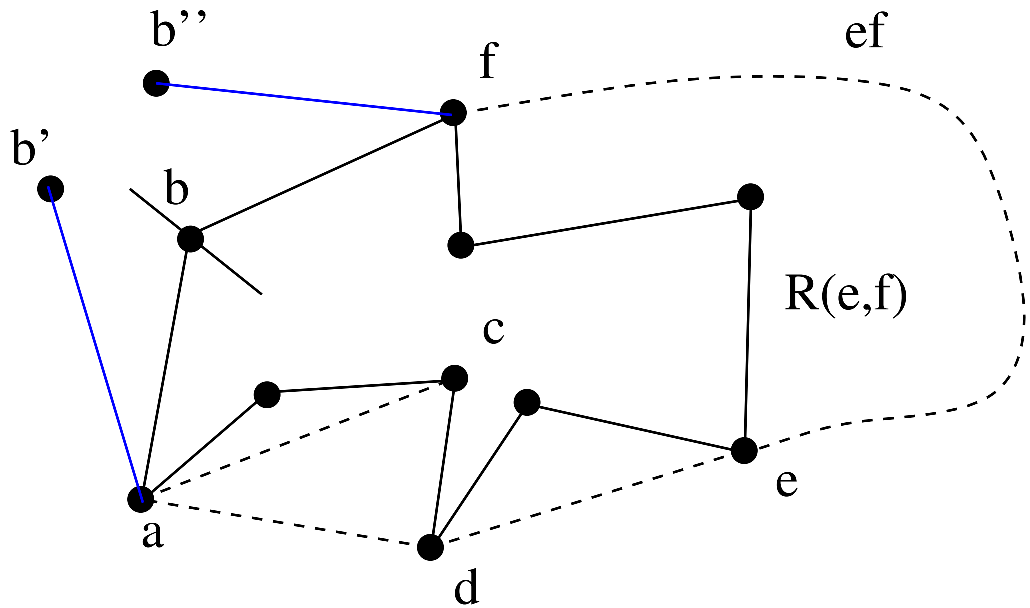
**Theorem:** For planar graphs the above Shortcut algorithm is 16-competitive.

Proof:

- Charged edges! Shortcuts, jump edges, set $P$
- $2(2+\delta)|P|$ in total! Factor $2$ DFS forth and back!
- MST of G, $|OPT| \geq |MST|$
- Show $|P| \leq (1+2/\delta)|MST|$
- $2(2+\delta)|P| \leq 2(2+\delta)(1+2/\delta)|MST| \leq 2(2+\delta)(1+2/\delta)|OPT|$
- Minimize $2(2+\delta)(1+2/\delta)$, $\delta = 2$, 16!

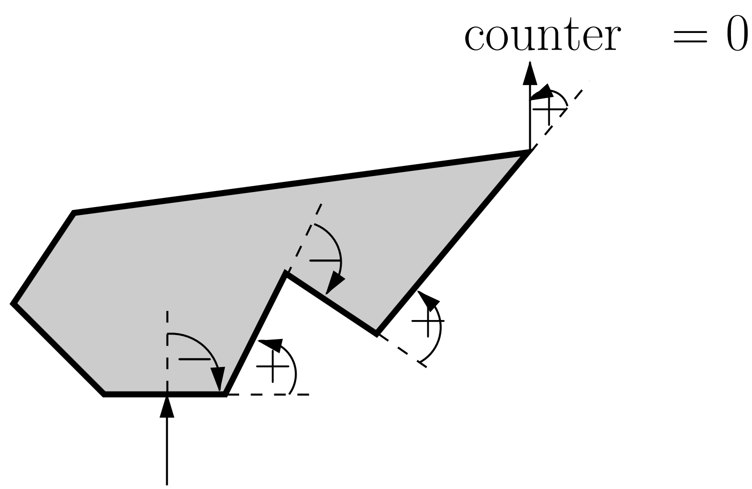
# Rep.: Shortcut algorithm/Analysis

- Proof: $|P| \leq (1 + 2/\delta)|MST|$
- Show: $|C| \leq |R|/\delta$, $C = MST \setminus P$
- Open the ring accordingly
- Process chords from *inside* to outside, budget $|R|$



# Rep.: Pledge Algorithmus

- Point-shaped agent/touch sensor
- Modi: Follow wall, follow a direction (exact)
- Single angluar counter



# Rep.: Pledge Algorithmus

1. Move into starting direction $\varphi$, until the agent hits an obstacle.
2. Rotate (right-turn) and follow the wall by Left-Hand-Rule.
3. Sum up the rotational angles until total total angular counter gets zero, then GOTO (1).

Possible errors: Counting angular rotations, hold the direction

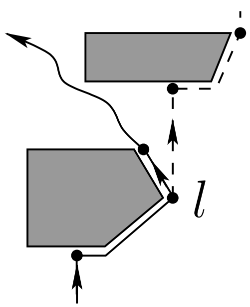
# Rep: Correctness error-free pledge

- **Lemma** Angular counter is never positive.
- **Lemma** In case of failure: Finite path $\Pi_\circ$ is repeated again and again.
- **Lemma** In case of failure: $\Pi_\circ$ has no self-intersections.
- **Theorem** Pledge finds an exit, if there is an exit.

$\Pi_\circ$ cw-order, Left-Hand-Rule, enclosed!

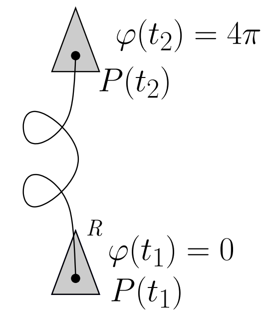
# Pledge algorithm with sensor errors

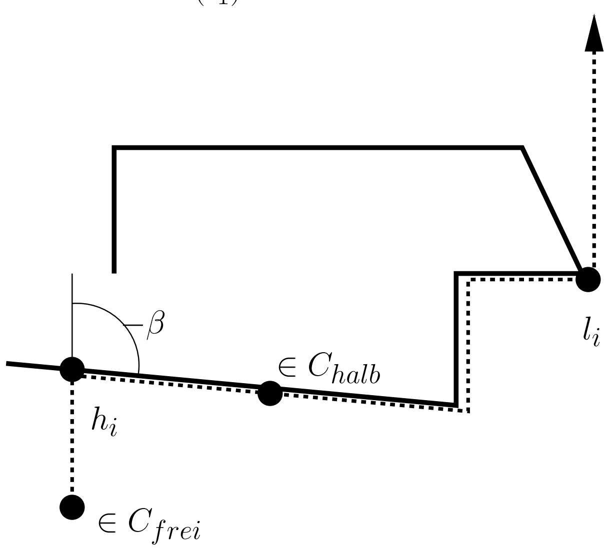
- Possible errors?
- Left-Hand-Rule, stable!
- Counting rotational angles!
- Hold the direction in the free space!
- For example: Compass!
- Full turns ok, but not precisely!
- Leave the obstacle slightly too early or too late!
- The main direction can be hold!
- Still correct?



# Notation/Model

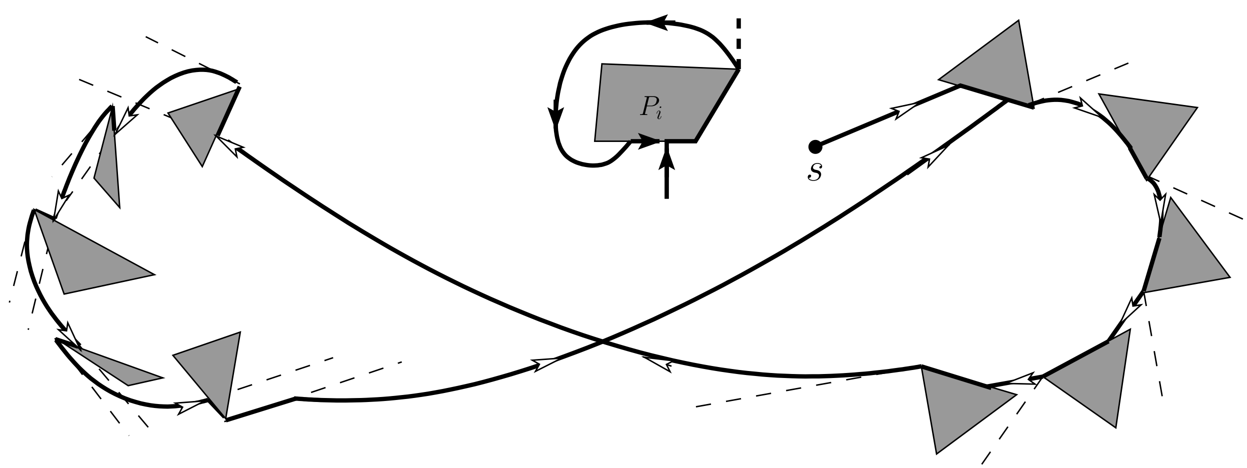
- Curves of the work-space
- Turning angle, position (ref. point):
  $C(t) = (P(t), \varphi(t))$
  with $P(t) = (X(t), Y(t))$
- For simplicity: point-shaped agent
- Hit-Point obstacle: $h_i$
- Leave-Point obstacle: $l_i$
- Boundary: $\mathcal{C}_{\text{half}}$, Free-Space: $\mathcal{C}_{\text{free}}$



# Typical errors!

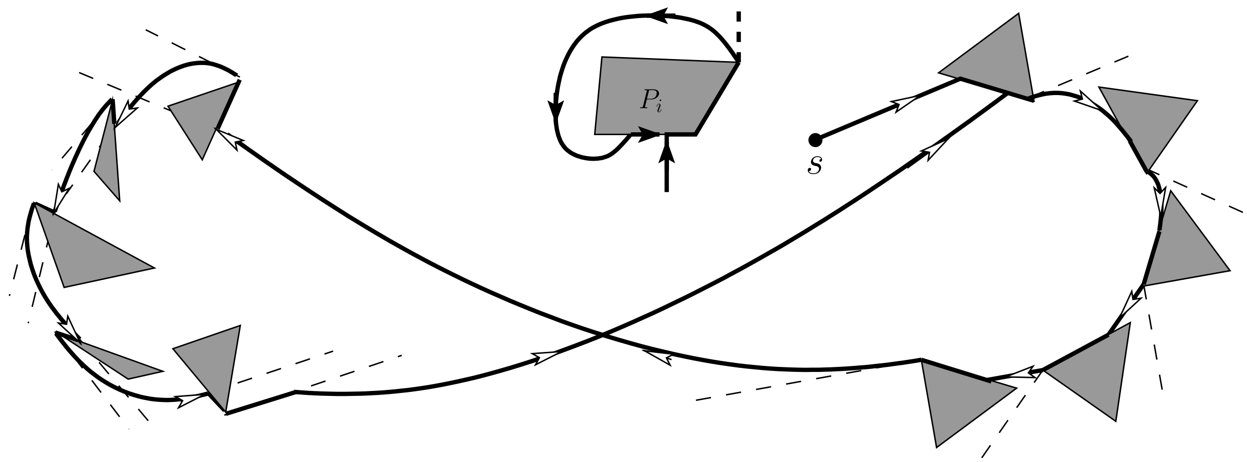
- Avoid infinite loop
- Leave into free space: Extreme direction error
- Or small errors sum up to large error
- Infinite loops!
- Condition: Leave direction has to be globally stable!



# Typical errors!

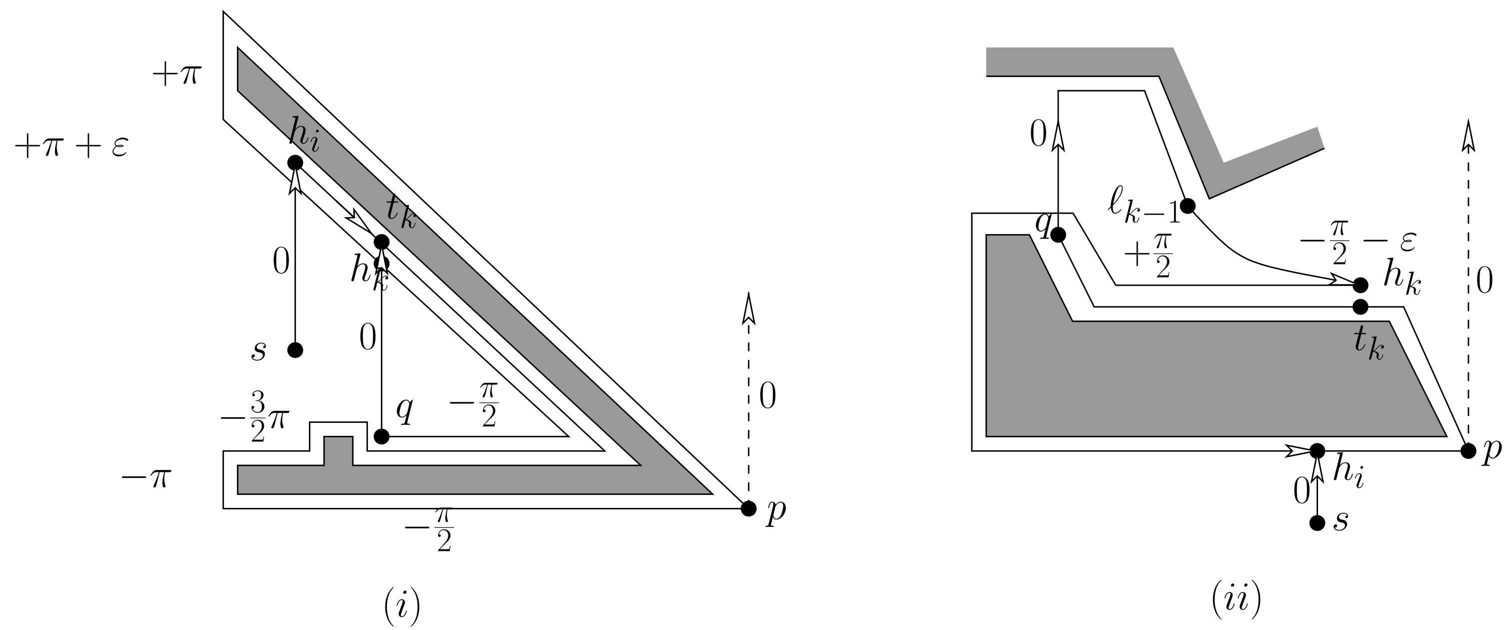
- Condition: Leave into direction $X$ has to be globally stable!
- $\mathcal{C}_{\text{free}}$-condition for the curve!

$$\forall t_1, t_2 \in C : P(t_1), P(t_2) \in \mathcal{C}_{\text{free}} \Rightarrow |\varphi(t_1) - \varphi(t_2)| < \pi$$



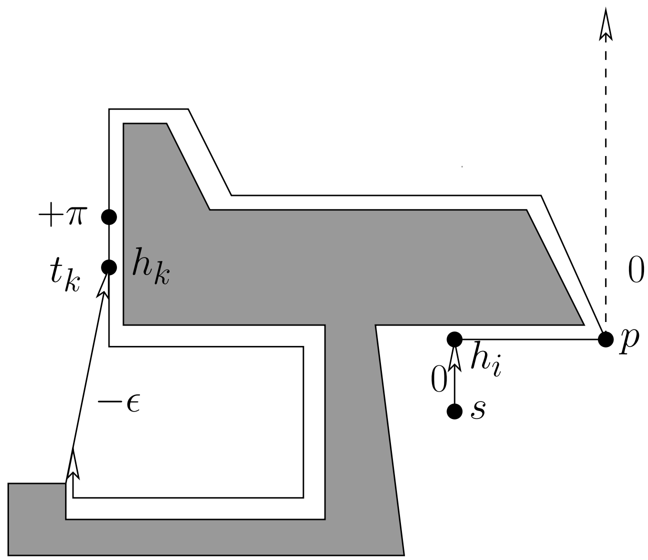
# Typical errors!

- $\mathcal{C}_{\text{free}}$-condition is not sufficient
- Overturn the angular counter locally at the obstacle!
- Infinite loops



# Typical errors!

- Do not overturn the counter locally
- $\mathcal{C}_{\mathrm{half}}$-condition: $\forall h_i, t \in C : P(t) = P(h_i) \Rightarrow \varphi(t) - \varphi(h_i) < \pi$



# Pledge algorithm with sensor errors

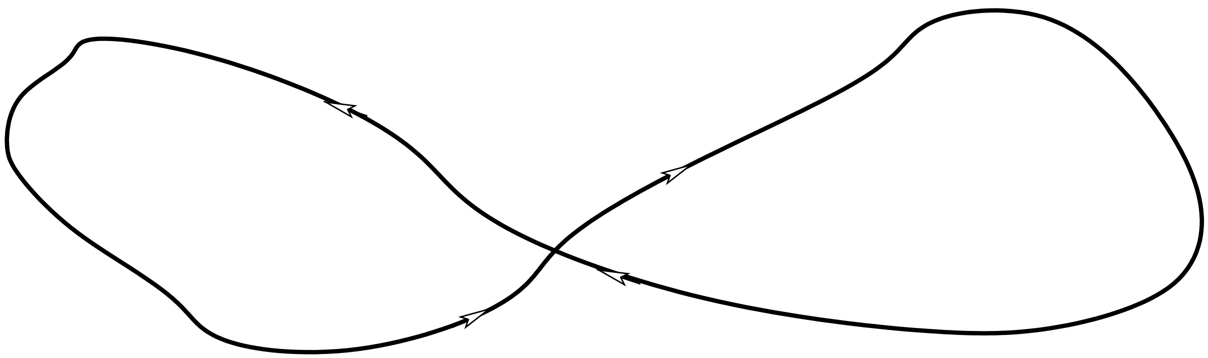
Pledge-like curve!

**Def.** $\mathcal{K}$ class of curves in $\mathcal{C}_{\mathrm{free}} \cup \mathcal{C}_{\mathrm{half}}$, with the following conditions:

1. Parameterized curve with turn-angles and position:
   $C(t) = (P(t), \varphi(t))$ mit $P(t) = (X(t), Y(t))$
2. Curve surrounds obstacel by Left-Hand-Rule
3. Leavs point is a vertex of an obstacle
4. $\mathcal{C}_{\mathrm{free}}$-condition holds:
   $\forall t_1, t_2 \in C : P(t_1), P(t_2) \in \mathcal{C}_{\mathrm{free}} \Rightarrow |\varphi(t_1) - \varphi(t_2)| < \pi$
5. $\mathcal{C}_{\mathrm{half}}$-condition holds:
   $\forall h_i, t \in C : P(t) = P(h_i) \Rightarrow \varphi(t) - \varphi(h_i) < \pi$

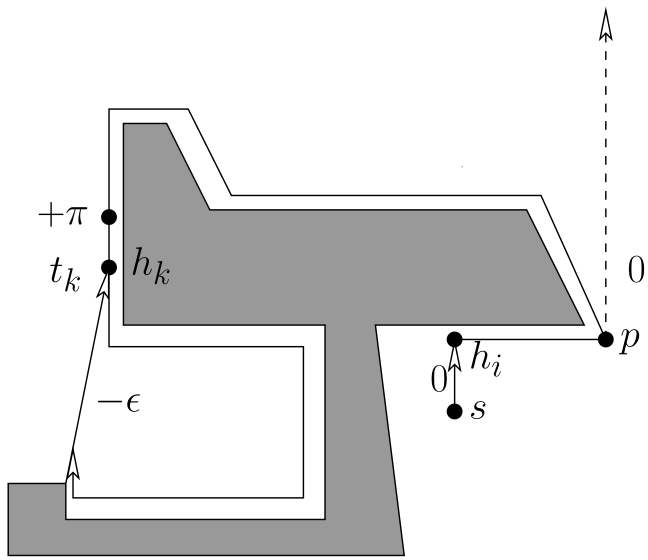
# Reminder: Error situation!

- Escape direction is globally stable!
- $\mathcal{C}_{\text{free}}$-condition:
  $\forall t_1, t_2 \in C : P(t_1), P(t_2) \in \mathcal{C}_{\text{free}} \Rightarrow |\varphi(t_1) - \varphi(t_2)| < \pi$



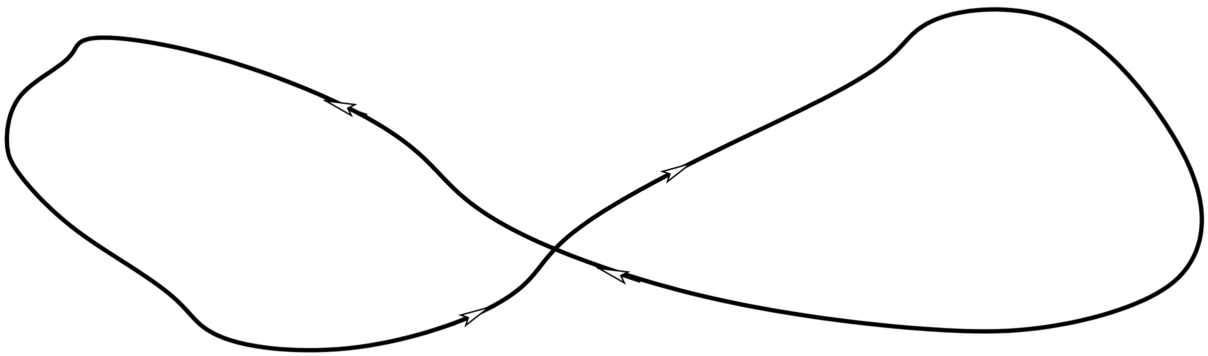
# Reminder: Error situation!

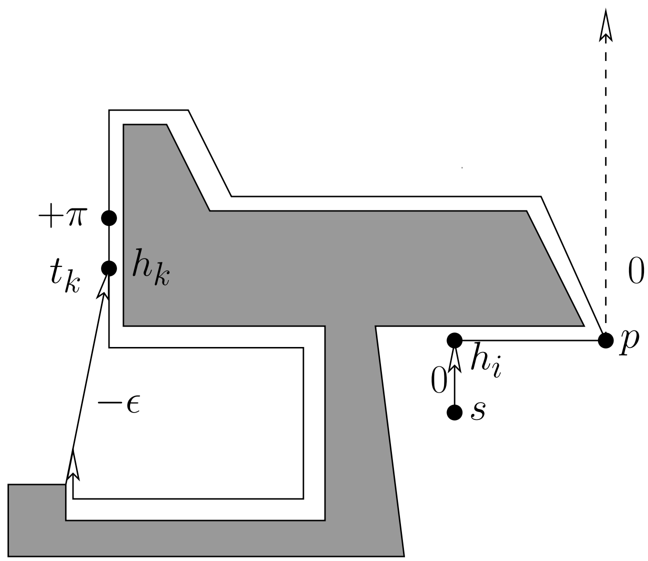
- Angular counter, no local overturn!
- $\mathcal{C}_{\mathrm{half}}$-condition: $\forall h_i, t \in C : P(t) = P(h_i) \Rightarrow \varphi(t) - \varphi(h_i) < \pi$



# Fulfill Curve-Definition: Hardware!

Compass with small deviation: Avoid situations!



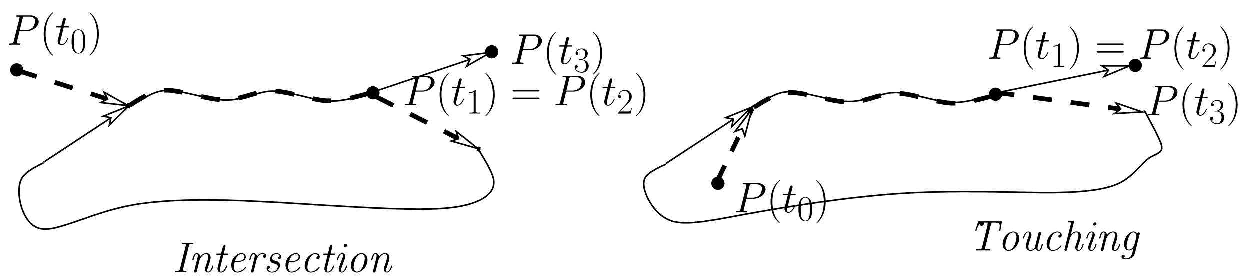


# Correctness proof!

**Lemma** A curve from $\mathcal{K}$ has no self-intersection.

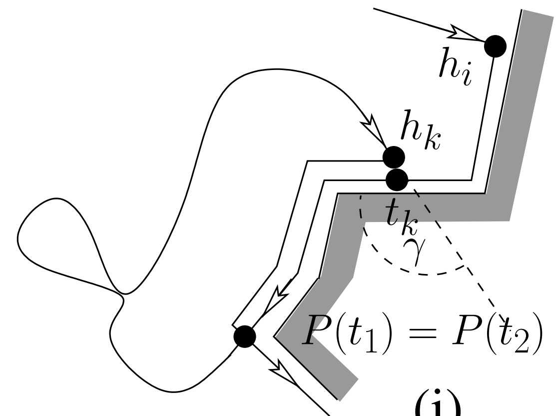
Proof:

- Assume: First crossing of $C$ by $t_1$ and $t_2$
- Single loop from $t_1$ to $t_2$: cw or ccw
- Case 1: Crossing in $\mathcal{C}_{\text{free}}$: Contradicts $\mathcal{C}_{\text{free}}$-condition!
- Case 2: Crossing in $\mathcal{C}_{\text{half}}$

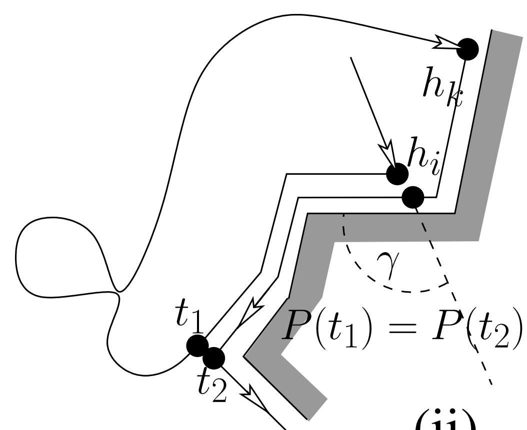


# Curves of $\mathcal{K}$, no self-intersection

- First loop: Enter at $h_i$, enter at $h_k$ again
- Intersection time $t_2$
- $P(h_k)$ also at $t_k$ with $h_i < t_k < t_1$, otherwise (ii) only touching
- $\varphi(h_k^+) = \varphi(h_k) + \gamma$ with $-\pi < \gamma < 0$
- From $t_k$ to $h_k^+$ full turn
- $\varphi(h_k^+) = \varphi(t_k) - 2\pi$
- $\varphi(t_k) - \varphi(h_k) < \pi$
- $\Leftrightarrow \varphi(h_k^+) + 2\pi - \varphi(h_k) = \varphi(h_k) + \gamma + 2\pi - \varphi(h_k) < \pi$
- $\Leftrightarrow \gamma < -\pi$, contradiction




(i)

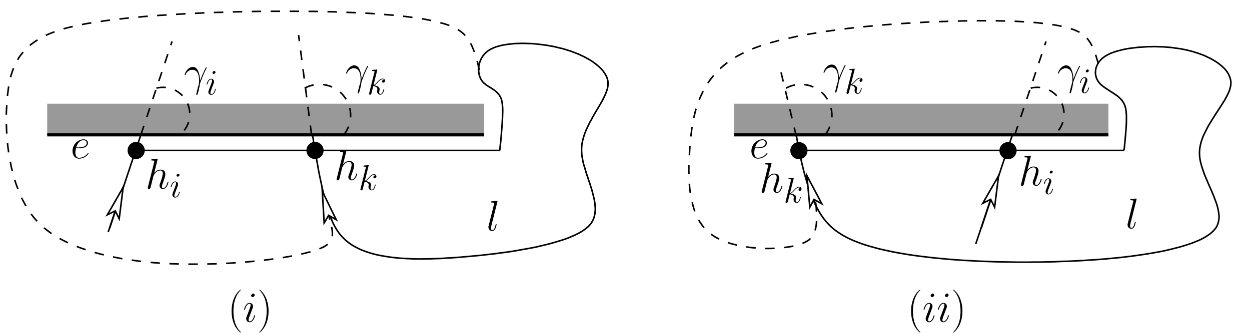



(ii)

# Correctness proof, sensor errors

**Lemma** A curve from $\mathcal{K}$ *hits* any edge only once.

- By contradiction! Assume $C$ hits $e$ twice
- Hit at $h_i$, then cw (or ccw) and again at $h_k$
- In $P(h_i)$,$P(h_k)$ with $-\pi < \gamma_i, \gamma_k < 0$ to $\varphi(h_i^+)$,$\varphi(h_k^+)$
- $h_i^+$ and $h_k^+$ follow edge $e$: $\varphi(h_k^+) = \varphi(h_i^+) + 2j\pi, j \in Z$
- Loop without intersection: Two cases $\varphi(h_k^+) = \varphi(h_i^+) \pm 2\pi$
- $|\varphi(h_k^-) - \varphi(h_i^-)| = | \pm 2\pi - \gamma_k + \gamma_i| > \pi$
- $\mathcal{C}_{\text{free}}$-condition does not hold!



# Correctness proof, sensor errors

**Lemma** For any curve from $\mathcal{K}$ we conclude: If the curve does not leave an obstacle any more, the obstacle encloses the curve.

Proof:

- Starting point free-space
- After the last hit, the curve fully surrounds the obstacle. Any round gives $\pm 2\pi$ to angular counter
- Positive? Compare to last hitpoint: $\mathcal{C}_{\mathrm{half}}$-condition
- $\mathcal{C}_{\mathrm{half}}$-cond.: $\forall h_i, t \in C : P(t) = P(h_i) \Rightarrow \varphi(t) - \varphi(h_i) < \pi$
- Therefore: $-2\pi$, Left-Hand-Rule, enclosed!

# Correctness proof, sensor errors

**Theorem** Any curve from $\mathcal{K}$ leaves a labyrinth, if this is possible.

- Starting-point free-space
- Assume: There is a successful path!
- **Lemma**: Has to leave any obstacle after a while!
- **Lemma**: Hit any edges only once!
- Finally the labyrinth will be left!

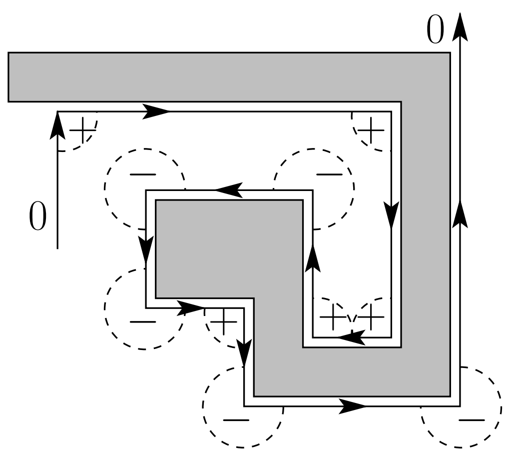
# Make use of a compass

**Corollary** By a compass with deviation less than $\frac{\pi}{2}$, any labyrinth will be left by a pledge like algorithm.

- Free-space angular range $(-\frac{\pi}{2}, +\frac{\pi}{2})$
- Direction deviates at most $\pi$!
- $\mathcal{C}_{\text{free}}$-condition holds!
- Along the boundary: Maximal overturn $+\frac{\pi}{2}$
- Free-space minimal $-\frac{\pi}{2}$
- Together: $\forall h_i, t \in C : P(t) = P(h_i) \Rightarrow \varphi(t) - \varphi(h_i) < \pi$ holds!
- $\mathcal{C}_{\text{half}}$-condition holds

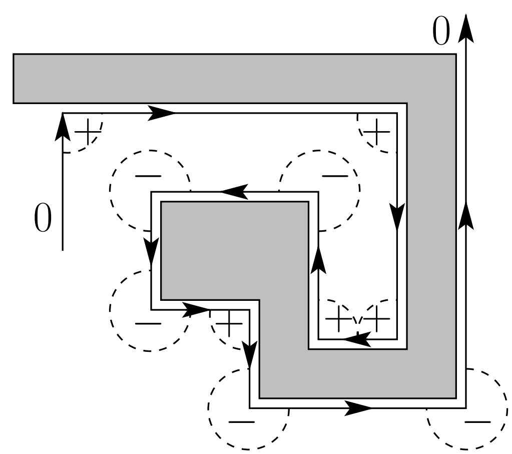
# Application! Rectangular scene!

- Scene allows roughly correct counting!
- Axis-parallel edges!
- Right-Turn, Left-Turn, count +1, -1, exact leave!
- Turning detected at the polygons!
- Free-Space: Deviation in $(-\frac{\pi}{2}, +\frac{\pi}{2})$
- Horizontal edge
- Vertical egde can be ignored: Slip along the edge!



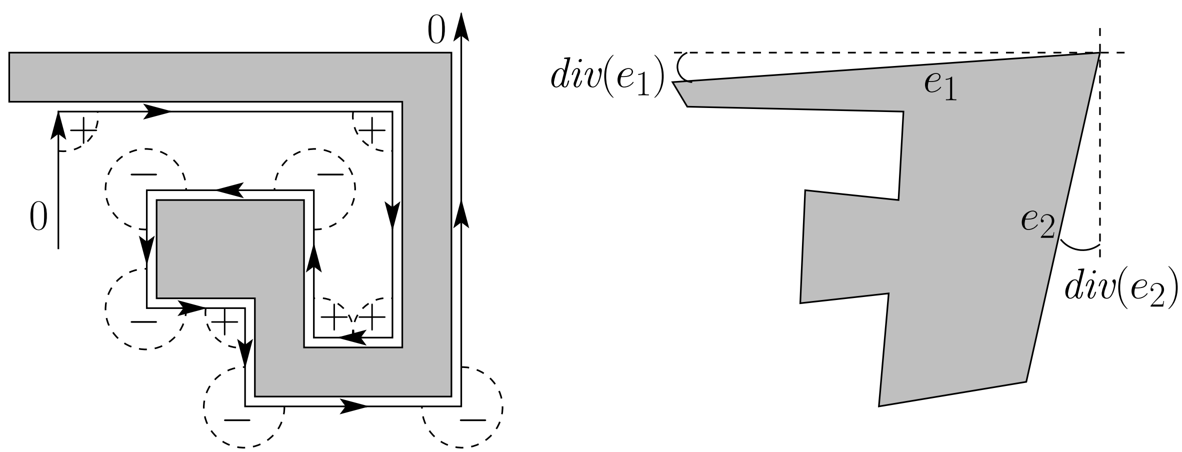
# Applications!

**Corollary** Axis-parallel environment, deviation in the free space within range $(-\frac{\pi}{2}, +\frac{\pi}{2})$, distinguish horizontal and vertical edges: Escape from a labyrinth!



# Deviations from axis-parallel: Pseudo orthogonal

- Small devaitions at the vertices! From global coordinates!
- 1. Condition: Numbers convex vert. $=$ reflex vert. $+ 4$
- Small deviations!
- div$(e)$ : $e = (v, w)$ smallest deviation from horizontal/vertical line passing durch $v$ und $w$
- $\mathrm{div}(P) := \max_{e \in P} \mathrm{div}(e) \leq \delta$, **Def.:** $\delta$-pseudo orthogonale Szene




(i) (ii)

# Szene $\delta$-pseudo orthogonal

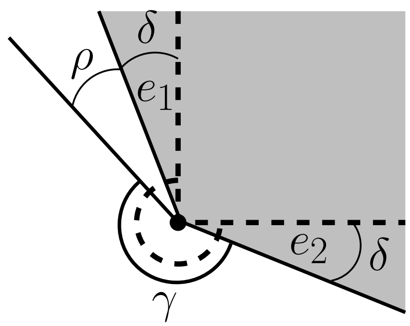
**Corollary** $\delta$-pseudo-orthogonal scene $P$. Measure angles with precision $\rho$ s.th. $\delta + \rho < \frac{\pi}{4}$. Deviation in the free space always smaller than $\frac{\pi}{4} - 2\delta - \rho$ from global starting direction. Escape from a labyrinth is guaranteed

1. Distinguish reflex/convex corners: Counting the turns!
2. Max. global deviation of starting direction: Intervall $\pi$
3. Distinguish: Horizontal/Vertical

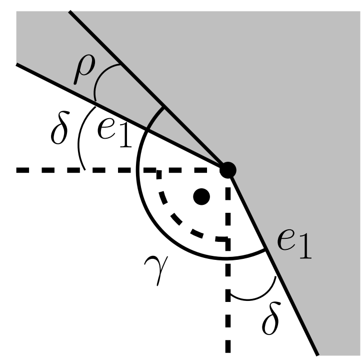
Proof: Blackboard!

# $\delta$-pseudo orthogonal scene

- Precision $\rho$ with $\delta + \rho < \frac{\pi}{4}$
- Free-space max. deviation $\frac{\pi}{4} - 2\delta - \rho$
- 1. Distinguish reflex/convex corners: Worst-case



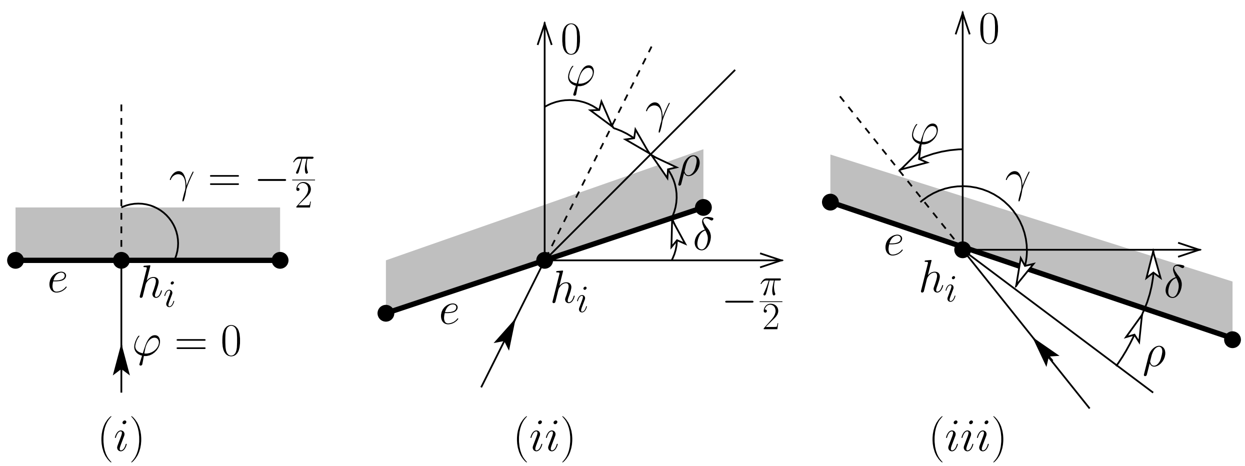

convex vertex




reflex vertex

# Szene $\delta$-pseudo orthogonal

- Precision $\rho$ with $\delta + \rho < \frac{\pi}{4}$
- Free-space max. deviation $\frac{\pi}{4} - 2\delta - \rho$
- 3. Horizontal/vertical: Worst-case



# Szene $\delta$-pseudo-orthogonal

- Precision $\rho$ with $\delta + \rho < \frac{\pi}{4}$
- Free-Space deviation $\frac{\pi}{4} - 2\delta - \rho$
- 2. Max. global deviation of starting direction: Intervall $\pi$
- Leave in $[-\delta, \delta]$
- Deviation for the next hit: $\frac{\pi}{4} - 2\delta - \rho$