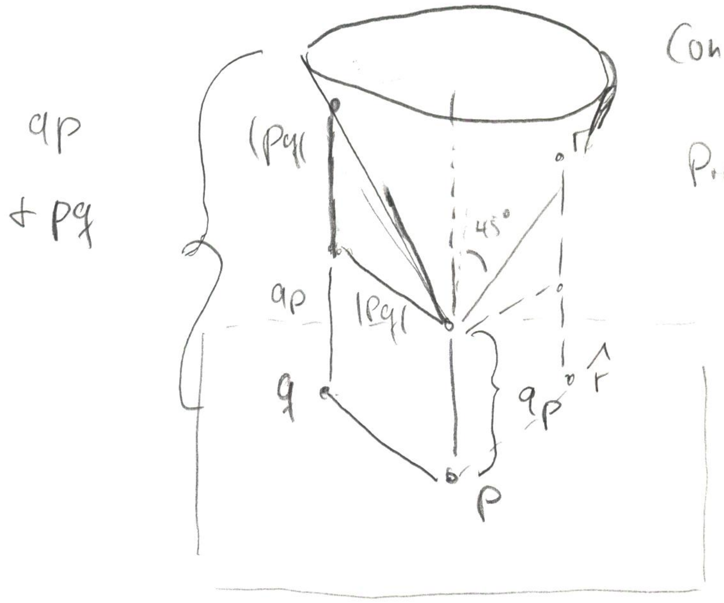
9

$$a_p := \frac{|C^p_{p_0}|}{\mathfrak{K}}$$



Cone k_p at height a_p over p!

Projection of points on k_p to the plane!

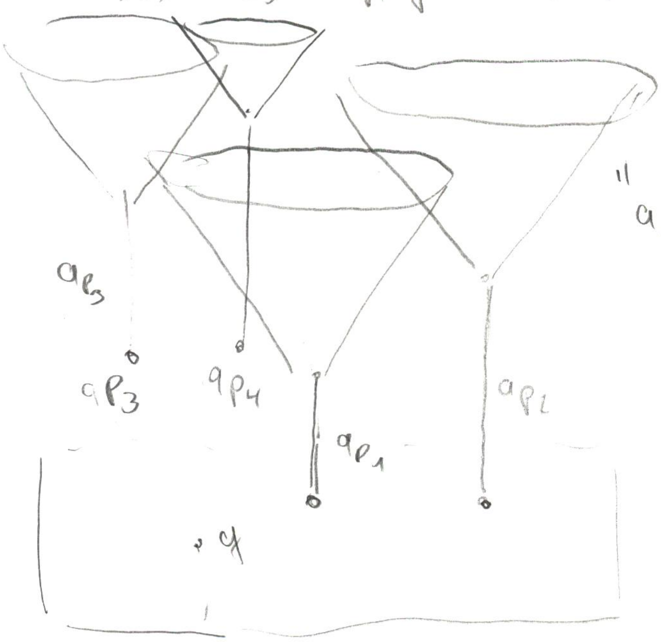
$$|r\hat{r}| = a_p + |p\hat{r}|$$

We have

$$(*) \quad \delta(p,q) \leq \mathfrak{K} \iff a_q \leq |pq| + a_p$$

$$\iff \text{point } (q_x, q_y, a_q) \text{ is below } k_p$$

This must hold for all vertices p before q!



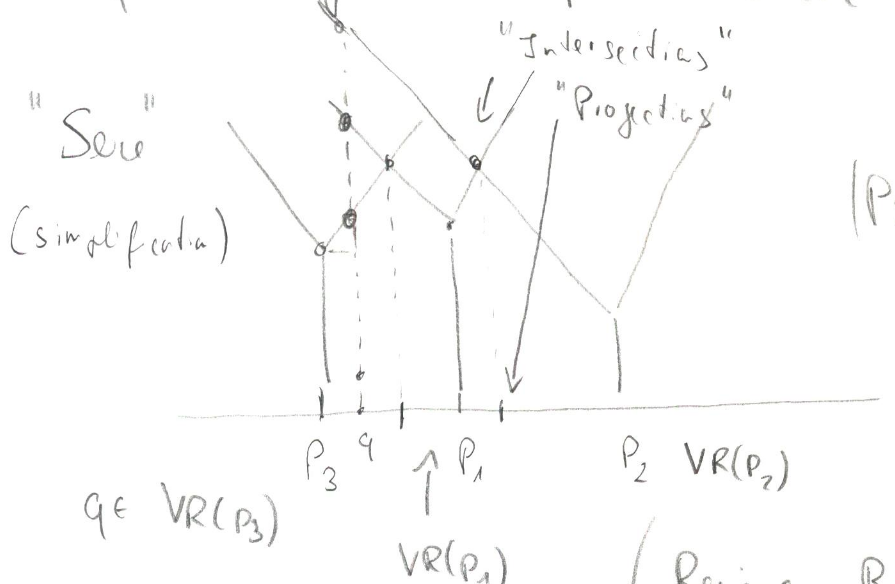
More cones!

"a slice" $\Longrightarrow$

(Simplification)

Voronoi-Diagramm in the plane, decomposition of cells of nearest neighborship.

$q \in \mathbb{R}^2$ belongs to site of the lowest cone



$|p_3 q| + a_{p_3} < |p_1 q| + a_{p_1}$

$< |p_2 q| + a_{p_2}$

line ℓ

(Regions, Boundary, Projections of Intersections)

Whole plane: Cones, lower envelope, additive weight a_p

$\boxed{\text{Black Box:}}$ Additive weighted Voronoi Diagramm AVD / AVR

Applet! $\rightsquigarrow$ Computed in $O(n \log n)$ Sweep

(*) For all p

$\left(\forall p: d(p,q) \le \mathcal{K} \iff (q_x, q_y, a_q) \text{ is below the lower envelope of the cones } K_p \right)$

$\iff q \in AVR(p_i)$ and

(q_x, q_y, a_q) is below K_{p_i}

Aufgabenstellung: Oriented chain $p_0, p_1, \ldots, p_i$

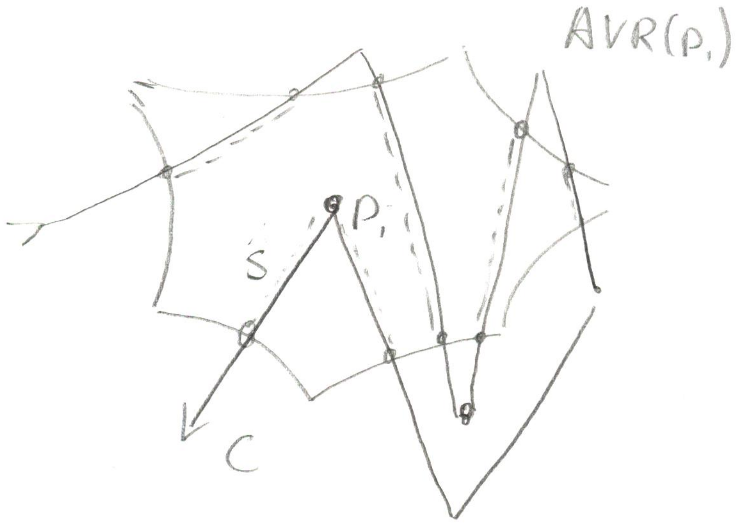
$q \in C$: Check $\partial(p,q) \le R$ for all p before q

- Localise q in $AVD(\{p_0, \ldots, p_i\})$
- If $q \in AVR(p_i)$, then check whether

$$(q_x, q_y, a_q) = \hat{q} \text{ is below } K_{p_i}$$

Problem: Test for all $q \in C$ not possible. Too many!

Solution: AVD Situation!



How! - Follow C in AVD (Trace!)

- For max. segments s check whether $\hat{s}$ is below K_{p_i}

Easy to do but $\Omega(n^2)$ segments s ?

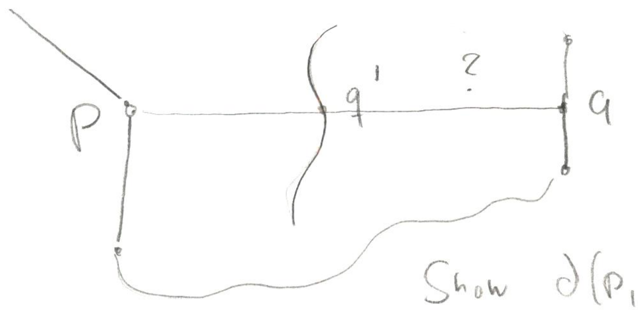
One edge crosses many Regions ?

Lemma 3 The maximum dilation of a chain C is attained for a pair (p, q) so that,

- p is a vertex of C
- p and q are mutually "visible". (Applet)

Proof: Let $\delta(C) = \delta(p, q)$

Assume that p does not see q.



Show $\delta(p, q')$ or $\delta(q', p) > \delta(C)$

$$\delta(C) = \frac{|C_p^q|}{|pq|} \overset{?}{\leq} \frac{|C_p^{q'}| + |C_{q'}^q|}{|pq'| + |q'q|} \quad \text{(Clear)}$$

$$\leq \max\left\{\frac{|C_p^{q'}|}{|pq'|}, \frac{|C_{q'}^q|}{|q'q|}\right\}$$

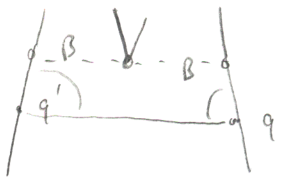
$$\leq \delta(C)$$

(Def Dilation)

$$\left(\frac{a+b}{c+d} \leq \max\left\{\frac{a}{c}, \frac{b}{d}\right\} \text{ for } a,b,c,d > 0\right)$$

$$\Rightarrow \quad \partial(c) = \max\left\{ \frac{|C_p^{q'}|}{|pq'|}, \frac{|C_{q'}^{q}|}{|q'q|} \right\}$$

Say $\dfrac{|C_{q'}^{q}|}{|q'q|}$

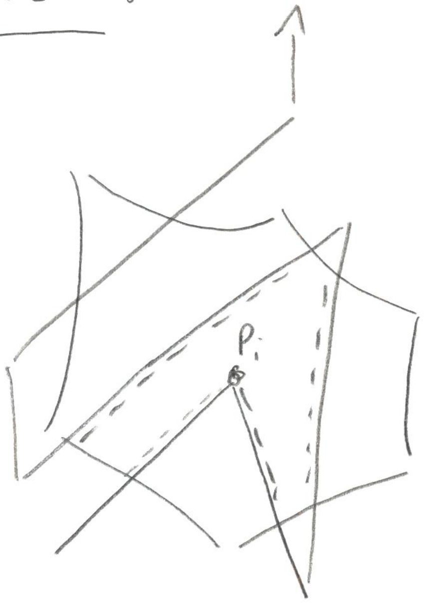


<u>Not a corna?</u>

Move until corna is not V!

Apply visibility argument again $\square$

Conclusion:



$VR(p_i)$

Only the visible segments S!

Black Box: Guibas et al. Combination lemma!

Compute the cells that contain p_i

Red - Blue Merge — Merged cells!

Chain ↑ ↑ Voronoi Diagramm

$(n = r + b + k)$

$O(n \log n)$ Running time
$O(n)$ Complexity

Theorem 3: Chain C and $k \geq 1$

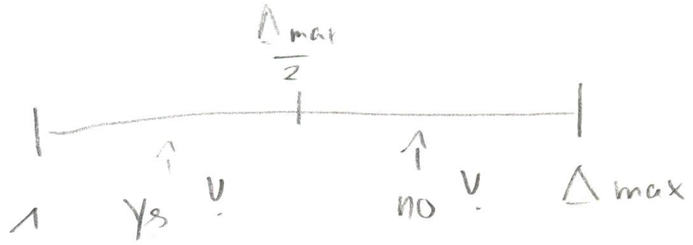
Answer for "$\partial(C) \leq k$?"

can be given in $O(n \log n)$ computation time

Proof: Just shown □ (with Black Boxes)

Now: Algorithm for computing $\partial(C)$

Binary search generalized!



Rough idea ✓

$d(c) \leq \frac{\Delta_{max}}{2}$?

Chan's randomized technique (very useful)

Based on the following idea:

How to find the minimum of r keys $A[1], A[2], \ldots, A[r]$?

Deterministic $O(r)$ ✓

Assume: Difference between Computation of $A[i]$ — Time C_0

Test of $A[i] \leq t$ — Time $D \ll C_0$

(Test is easy ✓)

Naive Algorithm

$t := \infty$;

For $i := 1$ to r do

If $A[i] < t$ then $t := A[i]$;

(Test D) (Comp. C_0) (*)

Return t;

Randomization: Choose a permutation $(i_1, i_2, \ldots, i_r)$ of $(1, 2, 3, \ldots, r)$

How often do we have to use (Comp) in the average ?

(*)

Argumentation:

(*) is used in step i

$\Longleftrightarrow$ $A[i]$ is minimum of $A[1], A[2], \ldots, A[i]$

Probability for that is $\frac{1}{i}$

By "Backward analysis" Randomization!

Assume we compute permutation as follows: (backwards)

Choose i_r uniformly at random from $\{1, \ldots, r\}$

Choose i_{r-1} uniformly at random from $\{1, \ldots, r\} \setminus \{i_r\}$

and so on!

Means that the probability that $A[i]$ is the minimum of $A[1], A[2], \ldots, A[i]$ is $\frac{1}{i}$

Any $A[1], A[2], \ldots, A[i]$ had the same chance to be choosen! (also the minimum) combinatorics

Randomized anlysis of the naive alg.

1. $O(rD)$ Tests (always)

2. $CO \times \sum_{i=1}^{r} \frac{1}{i} \leq (\ln r + 1)\, CO$ expected comp. time

Altogether: $O(rD + \log r\, CO)$

Generalization of this idea!

Problem Π, Instance $P \in \Pi$

Solution $w(P) \in \mathbb{R}$ Size of P : $|P|$.

P can be subdivided!

Lemma 4 (T. Chan)

Let $\alpha < 1$, $\varepsilon > 0$, $r \geq 1$ be constant.

Let D be a function so that $\frac{D(n)}{n^\varepsilon}$ increases monotonically

(Decision costs) (i.e. $D(n) = n \log n$ for $\varepsilon \leq 1$)

Assumption: For any $P \in \Pi$ we have

(i) for any $t \in \mathbb{R}$ we can answer the question $w(P) < t$ in time $D(|P|)$

(ii) we can find subproblems $P_1, \ldots, P_r \in \Pi$ so that

- $|P_i| \leq \alpha |P|$ for $1 \leq i \leq r$, $\alpha < 1$
- $w(P) = \min \{ w(P_1), \ldots, w(P_r) \}$

Then for any $P \in \Pi$ the solution $w(P)$ can be computed in expected time $O(D(|P|))$.