Computer Science Lecture

Discrete and Computational Geometry

Example questions!

Dilation

1. Give the precise definition of the notion of geometric/graph-theoretic dilation for a network!

2. How can we easily compute the dilation of a polygonal chain in polynomial time? Summarize the presented algorithms and the corresponding running times! Give upper and lower bounds on the running times! What kind of structural properties are helpful? How can we derive these properties?

3. Give a proof for the fact that there is a pair of points that attain the geometric dilation on a chain so that one point has to be a vertex.

4. Give a proof for the fact that some points that attain the maximum geometric dilation on a chain will be mutually visible!

5. Present the lower bound construction for computing the graph theoretic dilation of a chain! I.e, present the reduction of element-uniqueness!

6. Present the arguments for the fact that for a polygonal chain the dilation can be computed in expected O(n log n) time.

7. What is the conclusion of T. Chans method and how can we use it for computing the dilation of a polygonal chain. Summarize the steps of the algorithm.

Lattices and Minkowski's Theorem

1. Summarize Minkowski's Theorem. Give a precise sketch for the proof of the Theorem.

2. Present an example for the application of Minkowski's Theorem. What is the consequence for the problem of hiding in a (bounded) forest? How do we apply the Theorem in this case?

3. Summarize the Theorem of Dirichlet. How can we apply the Theorem to the general forest problem?

4. Formulate the two-square Theorem and present a sketch of the proof!

Incidence problems

1. Give a precise definition of the incidence problem between lines and points.

2. What does the Szemeredi-Trotter Theorem say?

3. Give lower and upper bounds on the number of I(m,n). Present the lower bound construction for m equals n, i.e., derive the lower bound.

4. Present the proof idea for the Szemeredi-Trotter Theorem by crossing numbers.

5. Summarize the crossing number Theorem and give a sketch of the proof.

6. Present the idea of how the crossing number Theorem is used for the upper bound on I(m,n) in the Szemeredi-Trotter Theorem.

Well-separated pair decompositions

1. Give the precise definition of a well-separated pair and present a figure that represents the definition. Give a precise definition of a WSPD.

2. Does a WSPD always exists? What is the main reason for using WSPDs?

3. Summarize the main Theorem of WSPD with respect to running time for construction and size of the resulting WSPD!

4. Present an algorithm for constructing a t-spanner out of a WSPD of size m. Give a proof that the construction is correct and show the relation ship between s and t.

5. Summarize the algorithm for constructing a WSPD efficiently. Explain the concepts of split-tree construction and the find-pairs procedure!

What is the reason for using partial split trees? Analyze the running time of the split-tree construction.

6. Show that the algorithm indeed constructs a WSPD! (Correctness!)

7. Give a sketch for the proof of the corresponding size of the WSPD. How do we bound the number of pairs reported by the algorithm? How can we derive the overall running time of the algorithm?

8. Present some application of the WSPD!

9. Give a proof for the running time of computing a closest pair by using a WSPD.

Brunns inequality

1. Summarize Brunns inequality and the Brunn-Minkowski Theorem.

2. Cutting three slices out of a bread. What does Brunns inequality imply?

3. Summarize the proof of Brunns inequality, if the Brunn-Minkowski Theorem is already given.

4. Give a sketch of the proof of the Brunn-Minkowski Theorem.

5. Explain the proof for brick sets A and B.

6. Explain why the d-th root in the function is necessary for the inequality.

Art gallery, VC-dimension and epsilon-net Theorem

1. Give a proof for the fact that \floor n/3 \floor agents are sufficient and somtimes necessary for guarding a simple polygon.

2. What are the main differences of guarding the boundary AND the whole polygon by a set of guards/a single guard? Give a proof or show an counterexample.

3. Give a precise definition of the VC dimension and the notion of shattering a subset for a set system F on a ground set X. Present some examples for the VC-dimension for some pairs (F,X).

4. Explain the notions of traversals and packings and give some examples for these definitions.

5. Give the precise definition of an epsilon net. What is the difference to a transversal? Present examples of epsilon nets of finite or infinte size.

6. Summarize the statement of the espilon net Theorem and show its application for the art gallery problem.

7. What is the relationship between the VC-dimension and the shatter function?

8. Summarize the shatter function lemma and give a sketch of the proof of the lemma.

9. Give an overview of the proof idea of the epsilon net Theorem. Explain where the shatter function lemma is explicitly used within this proof. Explain where the constant C in the proof is actually calculated and explain the impact on the art gallery Theorem.

10. Summarize the art gallery Theorem and explain the impact of the VC dimension of visibility polygons.

Randomized Incremental construction: Trapezoidal Map

1. Explain the idea of constructing a data-structure of a trapezoidal map for point location.
2. Show how a segment will be inserted into the data structure.
3. Explain the expected running time and structure size analysis for the randomized incremental construction!
   - Expected size
   - Expected query time
   - Expected running time
4. Expecially explain the idea of the backward analysis!
5. Explain the idea of finding out that the the randomized construction has the desired properties with high probability!
6. Why did we make use of the history graph structure?
7. How man rounds are necessary to obtain the desired structure?
8. For a polygonal chain the construction was further improved: Explain the idea of Seidels $O(n \log^* n)$ algorithm and sketch the analysis.

Abstract Voronoi diagram

1. Define abstract Voronoi diagrams, describe the motivation, and list several examples.
2. Explain the notion of an admissable set of bisecting curves
3. Let $(S, \mathcal{J})$ be a bisecting curve system. Please prove that the following assertions are equivalent.
   - If p, q, and r are pairwise different sites in S, then $D(p,q) \cap D(q,r) \subseteq D(p,r)$ (Transitivity)
   - For each nonempty subset $S' \subseteq S$, $R^2 = \bigcup_{p \in s'} \overline{\mathrm{VR}(p, S')}$
4. Please argue that for checking an admissible system, it is enough to check all subset of 3 sites.