Discrete and Computational Geometry, SS 18
Exercise Sheet "3": Application Minkowskis Theorem
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Thursday 3rd of May**.
- *You may work in groups of at most two participants.*
- *You can hand over your work to our tutor Raoul Nicolodi in the beginning of the lecture.*

Exercise 19: Proof details Two-Squares-Theorem (4 Points)

1. For $p = 17$, present the corresponding values of q, a and b, i and j in the proof of the Two-Squares-Theorem (Theorem 11). Finally $p = a^2 + b^2$ for $a, b \in \mathbb{Z}$ has to be fulfilled.
2. Prove the following statement: For the factor ring $\mathbb{Z}_p$ for a prime p only $a = \overline{1}$ and $a = -\overline{1}$ gives a solution for $a^2 = \overline{1}$.
   (You can make use of the following statement: $p|ab \Rightarrow p|a$ or $p|b$.)

Exercise 20: Minkowskis Theorem (4 Points)

- Present an argument that the Minkowski Theorem (Theorem 7) actually says that 2 lattice points different from the origin will be inside the set C.
- Argue that the boundedness of the set C is not a necessary condition of Theorem 7. Give an example for an unbounded set C that fulfills the conditions of Theorem 7 for $\mathbb{R}^2$.

Exercise 21: Application of Minkowskis Theorem (4 Points)

Consider the regular (5×5) lattice around the origin. Calculate the required expansion (radius r) of the *trees* at the lattice points so that any line $Y = aX$ hits at least one of the *trees*. Do the calculation in the following ways:

1. Calculate the radius r directly and precisely by considering the corresponding circles and lines.
   (W.l.o.g. only two cases have to be considered!)
2. Make use of the Minkowski Theorem and compute a non-trivial radius r that fulfills the requirement.

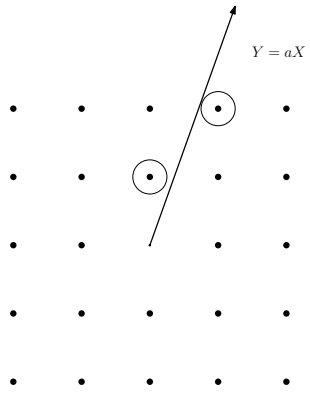

Figure 1: The regular (5×5) grid. The line passes the circles.