

Discussion: 06.06. - 08.06.

## Exercise Sheet 6

### Exercise 6.1: Vertex and chain maps

(4 Punkte)

A simplicial map  $f$  between simplicial complexes  $K$  and  $L$  maps each vertex of  $K$  to a vertex of  $L$  in such a way that the vertices of a simplex in  $K$  are mapped to the vertices of a simplex in  $L$ . Prove that

$g(\langle v_1 v_2 \dots v_d \rangle) := \langle f(v_1) f(v_2) \dots f(v_d) \rangle$  if all  $f(v_i)$  are different and  $:= 0$ , otherwise,

defines a chain map from the chain complex of  $K$  to the chain complex of  $L$ , and thus a homomorphism of the homology spaces.

*Hint: Ex.3 on sheet 1 might be useful*

### Exercise 6.2: Contraction vs collapsing

(4 Punkte)

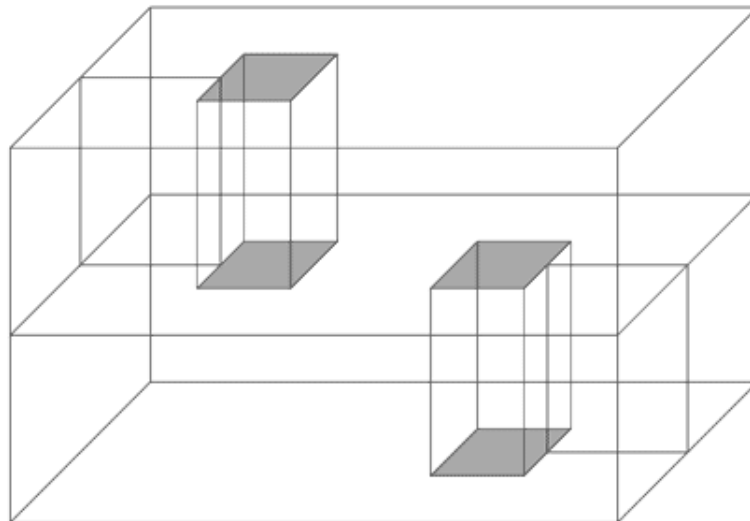
Which 1-dimensional complexes can be

- contracted into a point?
- collapsed into a point?

### Exercise 6.3: Bonus: House with two rooms

(4 Punkte)

An example for a contractible, but not collapsible 2-dimensional complex is given below by the "House with two rooms". Why is it not collapsible? Can you describe a deformation retraction to a point?



*Hint: It might be easier to imagine the retraction the other way around.*