Introduction to Computational Topology
Summer semester 2018

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UNIVERSITÄT BONN

Prof. Dr. Rolf Klein
Barbara Schwarzwald
Institute of Computer Science I

Exercise Sheet 7

Exercise 7.1: Collapsing a d-simplex (4 Punkte)

Consider the example of collapsing a solid 3-dimensional simplex to a single point as given in the lecture.

Prove that you can collapse any d-dimensional simplex to a single point by giving a description of the necessary sequence of deformations.

Exercise 7.2: Adding a simplex (4 Punkte)

In the spirit of incrementally computing β_p, there is another important method: Adding a simplex.

Let $K' = K \dot{\cup} \sigma$ from some simplex σ and let $\gamma := \partial'_k \sigma \in B_{k_1}(K', Q)$ with ∂ the boundary operator for K and ∂' the boundary operator for K'.

Prove the following theorem:

If γ has been a boundary (i.e. $\in \partial_k c_k(K, Q)$) already in K, then

$$\beta_p(K', Q) = \begin{cases} \beta_p(K, Q), & \text{for } p \neq k \\ \beta_k(K, Q) + 1, & \text{for } p = k \end{cases}$$

and else

$$\beta_p(K', Q) = \begin{cases} \beta_p(K, Q), & \text{for } p \neq k - 1 \\ \beta_{k-1}(K, Q) - 1, & \text{for } p = k \end{cases}$$

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Exercise 7.3: Fundamental groups of simple spaces (4 Punkte)

Determine and compare the fundamental group of the following simple topological spaces:

- $\mathbb{R}^2$, the euclidean plane
- S^2, the sphere
- $\mathbb{R}^2$ with a point removed
- S^2 with a point removed

Can you generalize your observation to the relation between the plane/sphere with k point removed?