Introduction to Computational Topology
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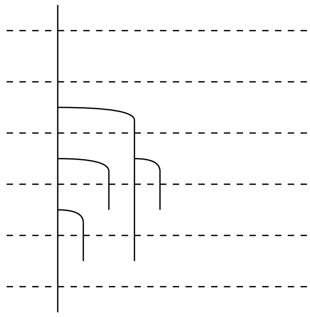
Exercise Sheet 8

Exercise 8.1: Elder Rule effects (4 Punkte)

Consider the following path decomposition of a merge tree generated from a filtration based on the elder rule - where the dashed lines symbolize the different levels of the filtration and a the bending path at an intersection is the path that ended upon two components merging.



How does the same graph look if instead of using the elder rule, a contrasting 'younger rule' is used? How many paths can span from a level a to a level b?

Exercise 8.2: Fundamental Lemma (4 Punkte)

Prove the following fundamental Lemma of persistent homology mentioned in the lecture:

Given the following definitions for a filtration $\emptyset = K_0 \subseteq K_1 \subseteq \ldots \subseteq K_n = K$

$$H_p^{i,j} := Imh_p^{i,j} = Z_p(K_i)/(B_p(K_j) \cap Z_p(K_i)) \subseteq H_p(K_j)$$

$$\beta_p^{i,j} := dimH_pi, j \qquad p^{th}\text{persistent Betti number}$$

$$\lambda_p^{i,j} := \text{number of p-classes born at } K_i \text{ and dying at } K_j,$$

so (since $\beta_p^{i,j-1} - \beta_p^{i,j}$ is the number of p-classes born at $\leq K_i$ and dying at K_j), while $\beta_p^{i-1,j-1} - \beta_p^{i-1,j}$ is the number of p-classes born at $\leq K_{i_1}$ and dying at K_j)

$$\lambda_p^{i,j} = \beta_p^{i,j-1} - \beta_p^{i,j} - (\beta_p^{i-1,j-1} - \beta_p^{i-1,j})$$

Then it holds for $0 \leq k \leq l \leq n$:

$$\beta_p^{k,l} = \sum_{i \leq k} \sum_{j < l} \lambda_p^{i,j}$$

Exercise 8.3: Complexity of pseudodisc polygon union (4 Punkte)

Consider the following situation.

n convex polygons (each of constant complexity) form a family of pseudodiscs, i.e. each pair of polygons have at most 2 intersections.

What is the complexity of the border of the union of this polygons?