

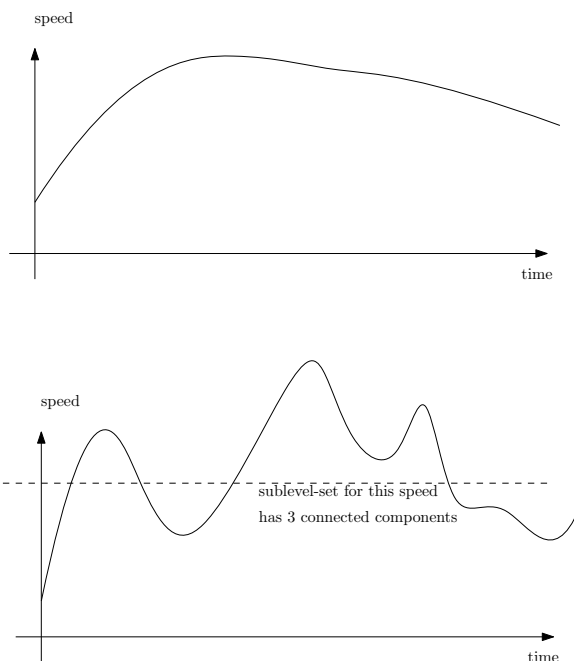
Discussion: 4.07. - 06.07.

## Exercise Sheet 9

### Exercise 9.1: Driver behaviour functions

(4 Punkte)

Analysis of topological filtrations have real-world applications. Consider the following two curves of driving speed over time for drivers on a highway - one of an experienced, relaxed driver and the other of a nervous driver.



Consider the filtration of the sublevel-sets and draw the according merge trees according to the elder rule. How can this be used to classify nervous or relaxed drivers?

### Exercise 9.2: Mini Ball Algorithm

(4 Punkte)

During the analysis of the running time for the MiniBall-Algorithm, we used a double recursive formula to analyse the runtime:

$$t_j(0) = 0 \quad \text{for all } j \qquad t_j(n) \leq t_j(n-1) + 1 + \frac{j}{n} t_{j-1}(n-1)$$

Prove that  $t_j(n) \leq (j+1)n$ .

### Exercise 9.3: Randomized Backwards Analysis

(4 Punkte)

Consider the standard algorithm for finding the maximum in an array with  $n$  integers, which consists of checking every entry in the array in order and keeping a *MaxSoFar* that holds the maximum found so far.

Let's say checking itself is a cheap operation, but updating *MaxSoFar* is expensive. What is the expected number of times *MaxSoFar* has to be updated, if the order of the items in the array is randomized?