## Online Motion Planning, WT 13/14 Exercise sheet 5 <br> University of Bonn, Inst. for Computer Science, Dpt. I

- You can hand in your written solutions until Tuesday, 26.11., 14:15, in room E.06.


## Exercise 13: Shortest $s$ - $t$-paths

(4 points)
We consider a rectangle $P$ as shown in Figure 1. Let $a$ and $b$ denote the


Figure 1: Rectangle $P$
width and height of $P$. Points $s$ and $t$ are centered at the low and high horizontal boundary edge of $P$. Furthermore, we are given two point sets $L=\left\{\ell_{1}, \ldots, \ell_{k}\right\}$ and $R=\left\{r_{1}, \ldots, r_{m}\right\}$ on the left and right vertical boundary edge of $P$. The points in the sets $L(R)$ are labelled such that point $\ell_{i}\left(r_{i}\right)$ is strictly higher than any other point $\ell_{j} \in L\left(r_{j} \in R\right)$, if $j<i$.
Prove that any shortest path in $P$ from $s$ to $t$, that visits every point in the set $L \cup R$ before ending at $t$, visits the points in $L$ in ascending order and also visits the points in $R$ in ascending order.

Please turn the page!

## Exercise 14: Existence of triangulations

Prove that if a simple polygon $P$ has at least 4 vertices, then in $P$ there exists a line segment connecting two non-adjacent vertices of $P$. Furthermore show that this implies the existence of a triangulation of $P$.

## Exercise 15: Visibility in arrangements of line segments (4 points)

a) Let $P$ be a simple polygon of $n$ vertices, and $s$ be a point in $P$. Prove that the visibility polygon of $s$ has a most $n$ boundary edges.
$b)$ Prove that the lower envelope of a given set of $n$ non-intersecting line segments consists of at most $2 n-1$ line segments.

