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Pearls of Algorithms

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Exercise sheet 2.3

Exercise 1 Complexity of Triangulations

Let $S \subset \mathbb{R}^2$ be a set of n points in general position i.e. with no four points on a common circle. Let $r = |S \cap \partial\text{ch}(S)|$ be the number of vertices on the boundary of its convex hull. Show that any triangulation of S consists of exactly $2n - 2 - r$ triangles and $3n - 3 - r$ edges.

Exercise 2 Complexity of Cyclic Polytopes

Let $d > 1$ and let S be a finite subset of the moment curve

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R}^d \\ t &\mapsto (t, t^2, \ldots, t^d). \end{aligned}$$

Show that all points of S belong to the boundary of the the convex hull of S, i.e. $S \subseteq \partial\text{ch}(S)$.

Exercise 3 Graph Embedding on the Moment Curve

Let $G = (V, E)$ be a graph and let

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R}^3 \\ t &\mapsto (t, t^2, t^3) \end{aligned}$$

be the moment curve in $\mathbb{R}^3$. Show that any reasonable embedding of G on f is crossing-free.

Exercise 4 Packing Points in a Square

Let $S = [0,1]^2$ be the unit square. Can we place ten points in S such that they are pairwise far to each other in terms of $\|p - q\| > \frac{\sqrt{2}}{3}$?