Discrete and Computational Geometry, WS1516
Exercise Sheet "4": Chan's Technique and Detours
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Wednesday 25th of November, 12:00 pm**.
- *There is a letterbox in front of Room E.01 in the LBH builiding.*
- *You may work in groups of at most two participants.*

Exercise 9: Constants in Chan's method (4 Points)

We would like to find out how large the constants in the main lemma of Chan's randomized technique might become. We refer to the application of computing the detour of a polygonal chain. For the detour-of-the-chain computation we choose $\alpha = \frac{1}{2}$ and $r = 4$ for a decomposition as suggested. If the decision algorithm takes $O(f(n))$ time, the randomized optimization algorithm takes $R \times f(n)$ expected time for a constant R. Please analyze what R would be in the following way.

1. Assume that the decision algorithm runs in $R' \times n \log n$ time.
2. Choose an ϵ so that the precondition of Chan's technique will satisfied, e.g, $\frac{n \log n}{n^\epsilon}$ monotone increases in n and $(\ln r + 1)\alpha^\epsilon < 1$.
3. How many recursion steps l have to be done for your choice of ϵ?
4. Express constant R in terms of precise values of l, α and r and the variable parameter R.

Exercise 10: The Decomposition of a Polygonal Chain (4 Points)

Consider a polygonal chain C with n polygonal vertices, and let V be the set of polygonal vertices of C. For any two points $p, q \in C$, the detour $\delta_C(p, q)$ between p and q in C is $\frac{|C_p^q|}{|\overline{pq}|}$, where C_p^q is the simple path between p and q in C, and the detour δ_C of C is $\max_{p,q \in C} \delta_C(p, q)$. Let W be a subset of V, and let Q be a subchain of C. Furthermore, Let $\delta_C(W, Q)$ be $\max_{p \in W, q \in Q} \delta_C(p, q)$, and let $\delta_C^*(W, Q)$ be $\sup_{(p,q) \in W \times Q, \overline{pq} \cap Q = \emptyset} \delta_C(p, q)$.

- Please give an example in which there exists a pair of points, $p \in W$ and $q \in Q$ such that $\delta_C(p, q) = \delta_C(W, Q)$ but $\overline{pq}$ intersects C.
- please prove that if $\delta_C(W, Q) = \delta_C$, $\delta_C(W, Q) = \delta_C^*(W, Q)$.