Discrete and Computational Geometry Winter term 2016/2017
Exercise Sheet 12
University Bonn, Institute of Computer Science I

Deadline: Tuesday 31.1.2016, until 12:00 Uhr | Discussion: 6.02. - 10.02.

- *This exercise sheet is NOT relevant to the completion of the exam admission, but covers the topics of the current week for completeness. Hence, submission is optional.*
- *In case you want to submit to get your solutions checked, please give your solutions directly to the tutor or put them in the postbox at LBH next to E.01 until the deadline. Write your names well visible and readable on the first page. If your solutions consists of multiple pages, make sure they are well connected.*
- *It is possible to submit in groups of up to three people.*

Aufgabe 1: Golden ratio properties (4 Points)

One definition of the golden ratio is as follows: Two quantities a and b with $a > b$ are in the golden ratio φ if $\frac{a}{b} = \frac{a+b}{a}$. Then $\varphi = \frac{a}{b}$.

Prove the following:

a) For the golden ratio φ it holds: $\varphi^2 - \varphi = 1$.

b) Let ABCDE be a pentagon (not necessarily regular), such that each diagonal cuts off triangles of area 1 from the pentagon. Then show, that the intersections of the diagonals divide each in the golden ratio.

Aufgabe 2: Closed forms of recursive functions (4 Points)

Consider the following recursive formula: $f(0) = 0$, $f(1) = 1$, $f(x) = 2 * f(x-1) + f(x-2)$

Find a closed form with a method of your choice (matrix trick, generating functions).

Aufgabe 3: L1-Embedding (4 Points)

In the lecture we proved that for any finite point set $S \subset \mathbb{R}^n$ with $|S| = n$ and any $\varepsilon > 0$ it is possible to find a $(1+\varepsilon)$-embedding $f : S \to \mathbb{R}^k$ of limited dimension $k \in O(\frac{1}{\varepsilon^2} \log n)$. For this we considered the euclidean metric $|.|$ for both spaces.

Is this also possible when considering the L_1-metric for S but the euclidean metric for the image space $\mathbb{R}^k$?

In other words: Can we find for each S a (large) $N \in \mathbb{N}$ and an according *isometric* embedding in the euclidean R^N in this variant? Or is there a finite S in the metric space $(\mathbb{R}^n, \|.\|_{L_1})$ and a constant $C > 1$, such that for each N each embedding $f : S \to (R^N, |.|)$ has at least dilation C?

Prove your answer!