Discrete and Computational Geometry Winter term 2016/2017
Exercise Sheet 05
University Bonn, Institute of Computer Science I

Deadline: Tuesday 22.11.2016, until 12:00 Uhr Discussion: 28.11. - 2.12.

- *Please give your solutions directly to the tutor or put them in the postbox at LBH next to E.01 until the deadline. Write your names well visible and readable on the first page. If your solutions consists of multiple pages, make sure they are well connected.*
- *It is possible to submit in groups of up to three people.*

Aufgabe 1: Trapezoidal Decomposition Procedure (4 Points)

Apply the procedure to create a trapezoidal decomposition described in the lecture. Given the following point set S in $\mathbb{R}^2$.

$$S = \{(-4,-4), (0,-3), (0,3), (3,-6), (6,1)\}$$

a) Create (per Hand) the euclidean Voronoi Diagram $V(S)$ of S.

b) Create (per Hand) the trapezoidal decomposition $T(V(S))$, such that you can locate a query point p from $[-10,10] \times [-10,10]$ in the voronoi regions.

c) Create a DAG $D(V(S))$ for $T(V(S))$ as a data structure for point localisation using the incremental procedure from the lecture. Add the line segments of the Voronoi Diagram from the left to the right (ordered by the x-coordinate of their left endpoint, breaking ties by x-coordinate of the right endpoint).

d) Mark the query path for the following 3 queries in $D(V(S))$:
$p_1 = (1,1)$, $p_2 = (9,-8)$, $p_3 = (-1,0)$.
Which voronoi regions do you get as a result?

Aufgabe 2: Trapezoidal Decomposition Properties (4 Points)

Prove the following propositions:

a) Every face f of a trapezoidal decomposition $T(S)$ of a set S of n line segments in general position[1] is bordered by one or two vertical edges and exactly two non-vertical edges.
Tip: First prove that every f is convex.

b) The trapezoidal decomposition $T(S)$ of a set S of n line segments in general position[1] consists of at most $6n+4$ vertices and at most $3n+1$ trapezoids.

[1] The line segments are in general position, if they intersect only at endpoints and no two different endpoints (from the same or different line segments) have the same x-coordinate.

Aufgabe 3: Log* (4 Points)

Consider $\log^* n := \min\left\{m \in \mathbb{N}_0 \;\middle|\; \log_2^{(m)}(n) \leq 1\right\}$ [1]

where the m-fold application of $\log_2$ on n is denoted as $\log_2^{(m)}(n)$, i.e.

$$\log_2^{(m)}(n) := \begin{cases} n & \text{falls } m = 0 \\ \log_2(\log_2^{(m-1)}(n)) & \text{sonst} \end{cases} .$$

a) Prove that $\log^* n$ is the smallest number $m \in \mathbb{N}_0$, such that the tower from m-many twos $\text{tower}(m) := 2^{(2^{\cdot^{\cdot^{2}}})}$ has value at least n.

b) What is the smallest number $n \in \mathbb{N}$ with $\log^* n = 5$?

c) Prove $\log^* n \in O(\log n)$!

[1] In the lecture we defined $\log^* n$ as $\max\left\{h \in \mathbb{N}_0 \;\middle|\; \log_2^{(h)}(n) \geq 1\right\}$. However both definitions differ only by ± 1, which doesn't matter in applications. This exercises reads much nicer this way.