Discrete and Computational Geometry, WS1415
Exercise Sheet "5": Dynamic Settings and Chan's Randomized Technique
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 18th of November 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 13: Dynamic Setting for Trapezoidal Decomposition (4 Points)

Consider a set N of n segments, let π be a random permutation $(S_1, S_2, \ldots, S_n)$ of N, and let $H(\pi)$ be the history graph with respect to π, i.e., the set of trapezoids created during the incremental construction following π. Assume we swap the orders of S_i and S_{i+1} to get a new permutation $\pi' = (S_1, \ldots, S_{i-1}, S_{i+1}, S_i, S_{i+2}, \ldots S_n)$, and let $H(\pi')$ be the history graph with respect to π'.

1. Please describe the difference between $H(\pi)$ and $H(\pi')$
2. Please analyse $|H(\pi) \oplus H(\pi')|$. ($A \oplus B$ means $A \setminus B \cup B \setminus A$).

Exercise 14: Random-Min (4 Points)

The following algorithm selects the minimum of r numbers, $A[1], A[2], \ldots, A[r]$.
Algorithm RAND-MIN

1. randomly pick a permutation $\langle i_1, \ldots, i_r \rangle$ of $\langle 1, \ldots, r \rangle$

2. $t \leftarrow \infty$

3. for $k = 1, \ldots, r$ do

4. if $A[i_k] < t$ then (*decision*)

5. $t \leftarrow A[i_k]$ (*evaluation*)

6. return t

Please derive the expected number of times that step 5 will be performed.

Exercise 15: Chan's Randomized Technique (4 Points)

Consider a function $T(\cdot)$ satisfying the following recurrence:

$$T(n) = (\ln r + 1)T(\lceil \alpha n \rceil) + O(D(n)),$$

where r, $\alpha < 1$, and $\epsilon > 0$ are constants and $D(n)$ is a function such that $D(n)/n^\epsilon$ is monotone increasing in n. Please prove that if $(\ln r + 1)\alpha^\epsilon < 1$, $T(n) \leq C \cdot D(n)$, where C is a constant depending on r, α, and ϵ.