Online Motion Planning MA-INF 1314
Introduction

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Motion planning for mobile agents

- Lecture: Tuesday/Thursday 12:30 to 14:00
- Exercise groups: Starting next week 19/20th
  Tuesday: 10-12 Wednesday: 14-16
- Sign in now
- Manuscript on the webpage
- Slides on the webpage
- Old manuscript (in german)?
- Exercises
- Today: Short intro, different topics, some examples

Motion planning categories

- Elektronic devices
- Mechanical devices
- Control/Process engineering
- Artificial Intelligence
- Softwareengineering
- ⋮
- Plans: Algorithmic
- Input: Geometry of the Environment

Geometric Algorithms

- Geometry of Agent and Environment
- Solve motion planning problem
- Main difference: Online/Offline
- Incomplete Information
- Correctnes? Efficiency?
- Lower/Upper Bound
- Structural Properties
- Groundtasks
- Algorithms paradigms
- Formal proofs

Example I: Simple polygon, exploration

- Simple room, no obstacles
- Agent starts at s, *sees* everythings, returns to the start
- 360 degree vision
- Modell: Point-shaped agent, simple polygons, visibility polygon,
- Optimal route: Shortest Watchman Route
- Offline: Polygon is fully known
- Online: Union of visibility polygons
- Offline: Optimal algorithm
- Applets: Offline algorithm versus Online algorithm

Example I: Online/Offline

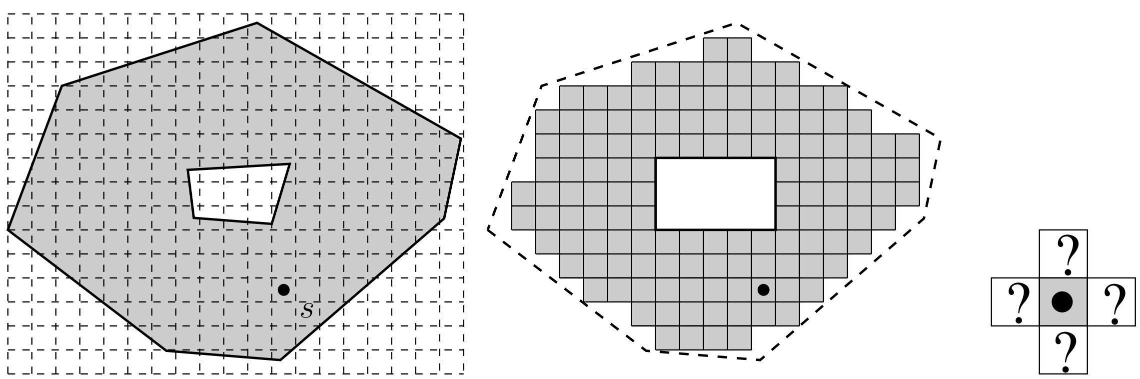
- Axisparallel polygon
- Offline Algorithm: O(n) Applet!
- Online, not optimal!
- Greedy Online Strategy!
- Proof: L_1-optimal!
- $\sqrt{2}$ Approximation!
- Competitive Ratio

Online motion planning: Modell

- Searching for a goal
- Exploration of an environment
- Process task on (subset of) environment
- Escape from a labyrinth
- Continuous/discrete vision
- Touch sensor/compass
- Build a map
- History: Simple $\Rightarrow$ complicated

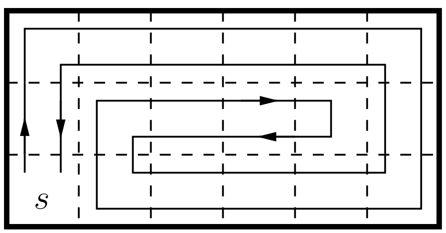
Example II: Exploration of grid-environments

- Grid world (cells), tool of cell-size, one-step from cell to cell
- Process any cell in the connected-component
- Touchsensor only detects neighboring cells
- Build a map, visit all cells, return to start
- Grid-explore applet

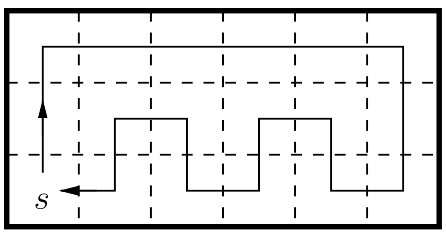


Example: Exploration of grid-environments

- Online DFS for cells $\Rightarrow$: Strategy
- Visit all connected cells: Correctness
- Number of steps: $2 \times (C - 1)$
- Optimum: At least C
- Ratio: smaller than 2 times the optimum, Performance
- No better strategy! Lower Bound!!
- Provoke detour! Proof of this later!!!



DFS



Optimal

Competitive analysis, competitive ratio

Definition 1: Let Π be a problem class and S be a strategy, that solves any instance $P \in \Pi$.
Let $K_S(P)$ be the cost of S for solving P.
Let $K_{\text{OPT}}(P)$ be the cost of the optimal solution for P.

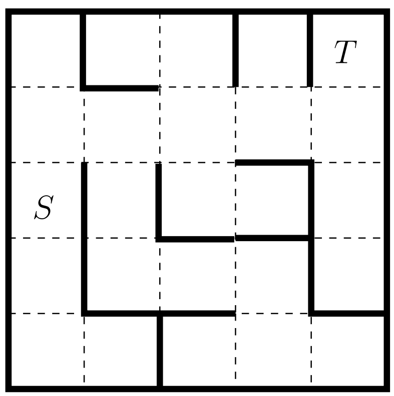
The strategy S is denoted to be c–competitive, if there are fixed constants $c, \alpha > 0$, so that for all $P \in \Pi$

$$K_S(P) \leq c \cdot K_{\text{OPT}}(P) + \alpha$$

holds.

Chap. 1: Labyrinths, Grids, Graphs

- Different 2D environments
- Tasks: Searching for a goal, escape, exploration
- Def: Labyrinth L intuitiv: Divide the plane by walls into corridors
- Def: Grid-Labyrinth grid/cell enviroment with walls on the edges
- History 1950: Shannon 5×5 Labyrinth with an elektr. Mouse

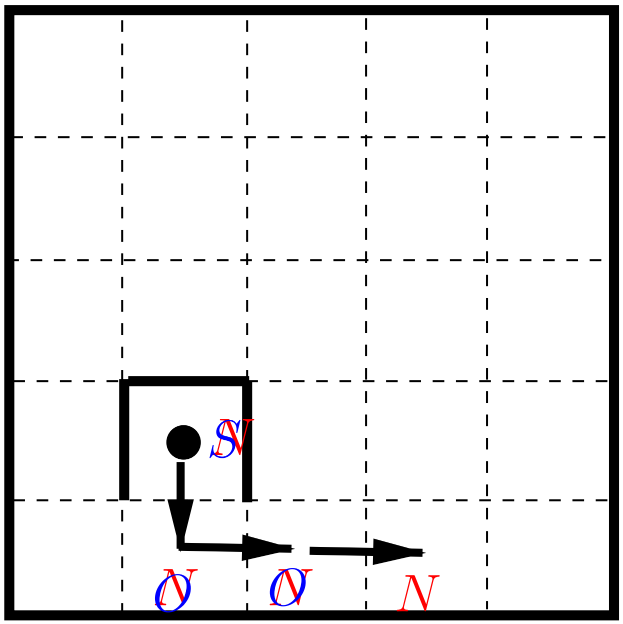


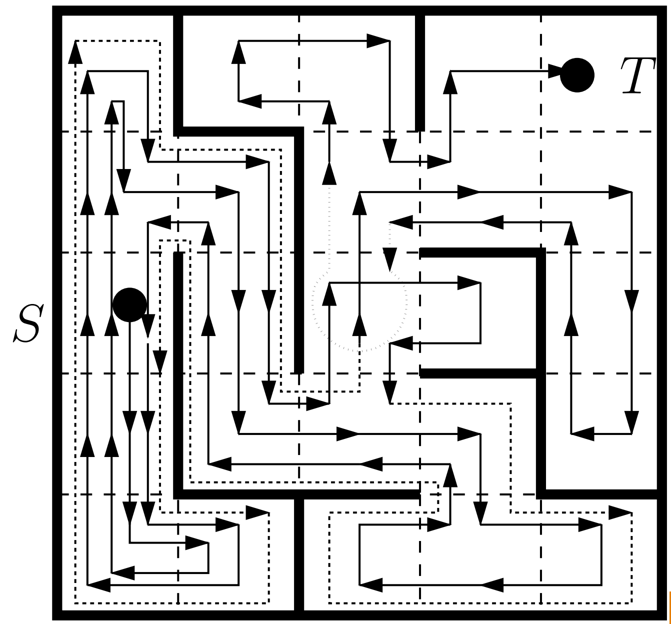
Chap. 1.1 Shannons Mouse Alg.

Historie 1950: Shannon 5×5 Labyrinth with an elektr. Mouse

- For any cell, there is a marker that stores the direction (N,E,S,W) in which the mouse left the cell at the last visit.
- Initialize any cell with marker N (north)
- While T has not been visited:
  Search for the first free cell in clockwise order starting from the current marker direction.
  Change the marker correspondingly and enter the corresponding cell.

Shannons Maus Alg.





Shannons Mouse: Correctness!

Theorem 1.1: Shannons Mouse Algorithm will always find the target in any grid labyrinth from any starting point, if a path from S to T exists.

Proof:

- Consider the strategy without the target
- Show: All reachable cells will be visited infinitely often
- Formal proof: Blackboard!

Efficient algorithm for graph-exploration

- Graph-exploration, visit all edges (and vertices)
- Vertex: Outgoing edges become visible! Build the full graph, return to the start!
- Visited edges can be located
- Strategy: Online-DFS for the edges, visits any edges twice

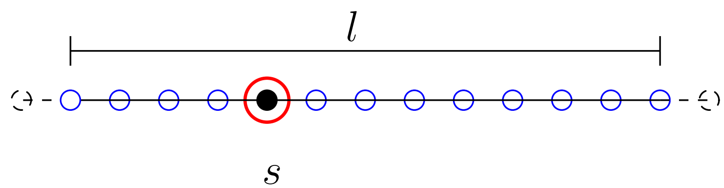
Theorem: Exploring an unknown graph requires roughly twice as many edge visits than the optimal exploration route for the known graph. DFS requires no more that twice as many edges.

Formal proof! Second part is already clear! Lower bound by worst-case adversary strategy

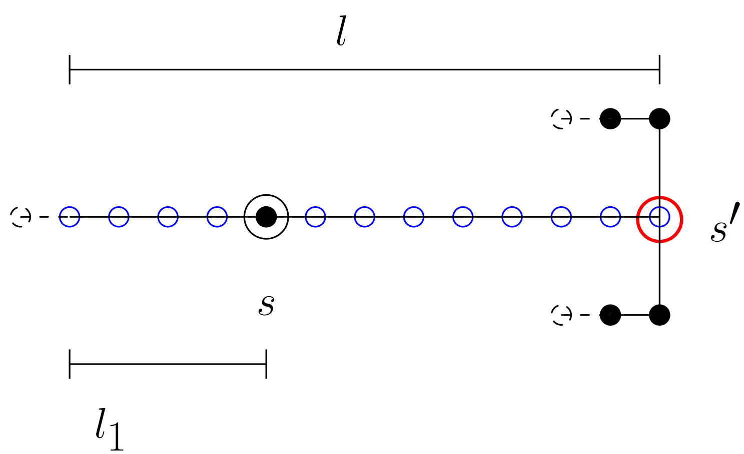
Graphexploration, Edge visits, Adversary

Adversary: $2 - \delta$ worse than the optimum

corridor, agent from s has explored l vertices

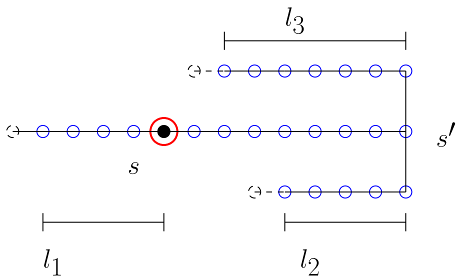


Now bifurcation at s'

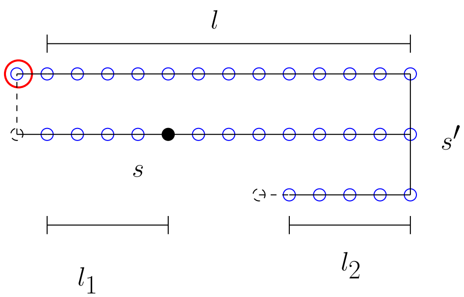


Graphexploration, Edge visits, Adversary

- Case 1: Agent returns to s



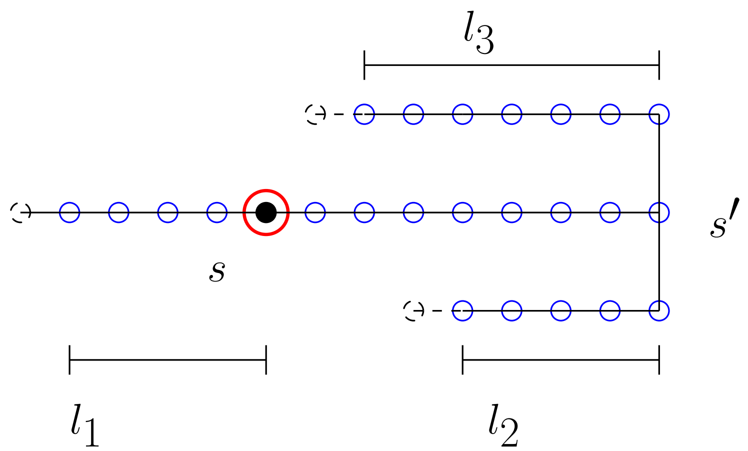
- Case 2: Agent reaches the end of the start corridor



- Close the corridors, present the scene

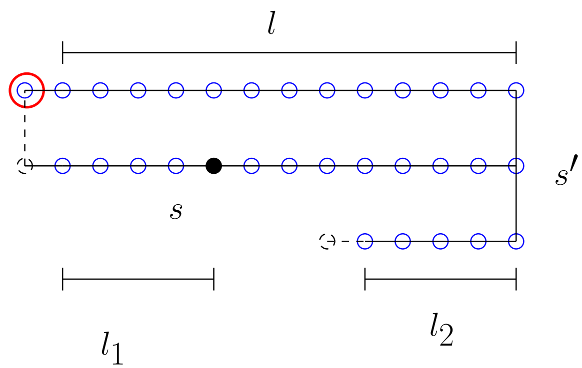
Adversary analysis: Case 1.

- Strategy S_{ROB} against S_{OPT}
- Case 1: Agent returns to s
- Up to now $|S_{\text{ROB}}| \geq 2l_1 + 2l_2 + 2l_3 + 2(l - l_1) = 2(l + l_2 + l_3)$
- From now on: $2(l + l_2 + l_3) + 6$
- Optimal: $|S_{\text{OPT}}| = 2(l + l_2 + l_3) + 6$
- $|S_{\text{ROB}}| \geq (2 - \delta)|S_{\text{OPT}}|$ for $l \geq \left\lceil \frac{3}{\delta} \right\rceil$



Adversary analysis: Case 2.

- Case 2: Agent reaches the end of start corridor
- Up to now: $|S_{\mathsf{ROB}}| \geq 2l_1 + l - l_1 + 2l_2 + l + 1$
- From now on: $l + 1 + 2(l_2 + 1) + l - l_1$
- Sum: $|S_{\mathsf{ROB}}| = 4(l + l_2) + 4$
- Optimal: $|S_{\mathsf{OPT}}| = 2(l + 1 + l_2 + 1) = 2(l + l_2) + 4$
- $|S_{\mathsf{ROB}}| \geq (2 - \delta)|S_{\mathsf{OPT}}|$ für $l \geq \left\lceil \frac{3}{\delta} \right\rceil$



Remarks

Corollary : DFS for Online-Exploration of graphs is 2-competitiv, there is no C-competitive strategy with $C < 2$.

- DFS is optimal
- Additive constant: Start situation
- Example: Goal very close
- Lower bounds only with arbitrary large examples

Remarks!

- Return to the target
- No return: Exercise
- Examples: Opt. Tour visits any edge twice
- Visit only the vertices, DFS for vertices? Exercise!