

Online Motion Planning MA-INF 1314

Polygon/Corner Exploration

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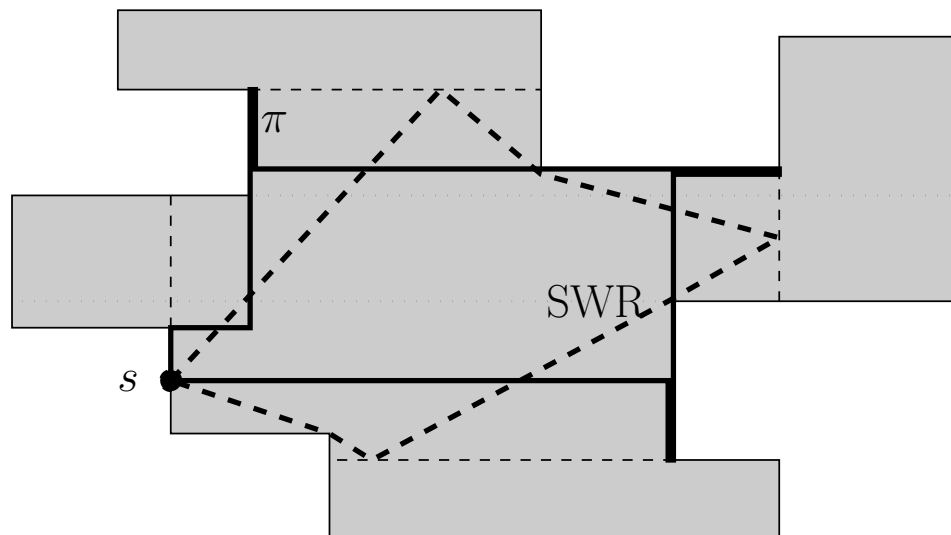
Rep: Exploration for the framework

Exploration rectilinear polygons DKP (Optimal $L_1/\sqrt{2}$ -comp)

WHILE Polygon not fully explored

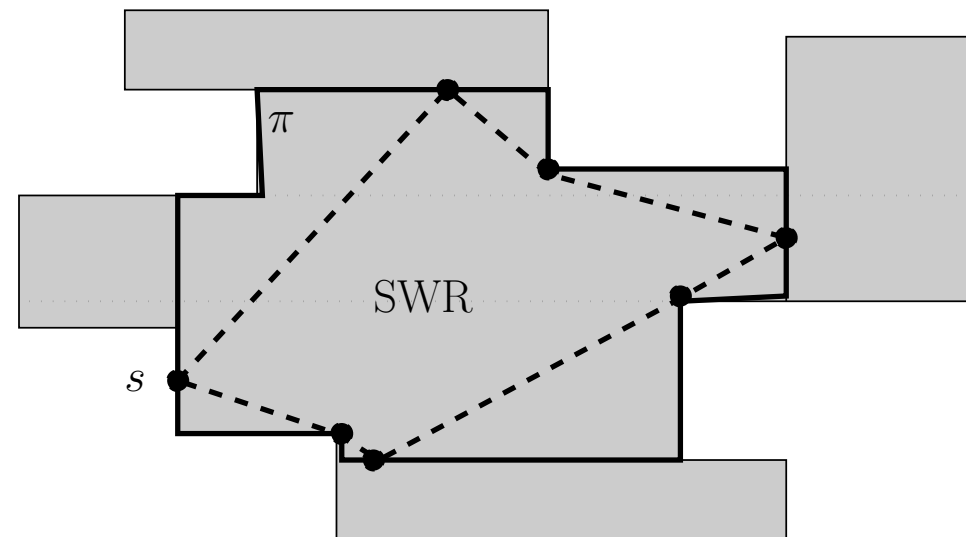
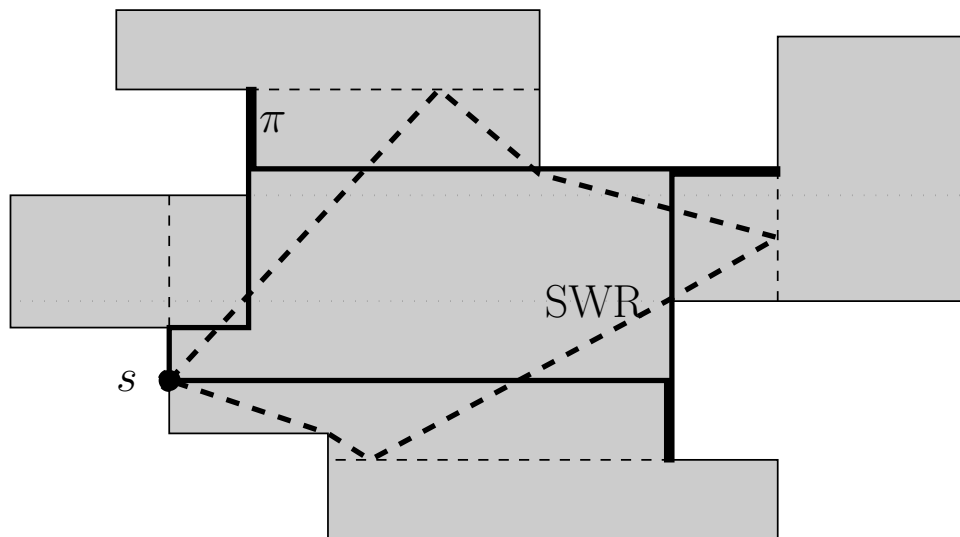
Do Move orthogonally toward the cut of next reflex vertex in cw-order along the boundary

END ■



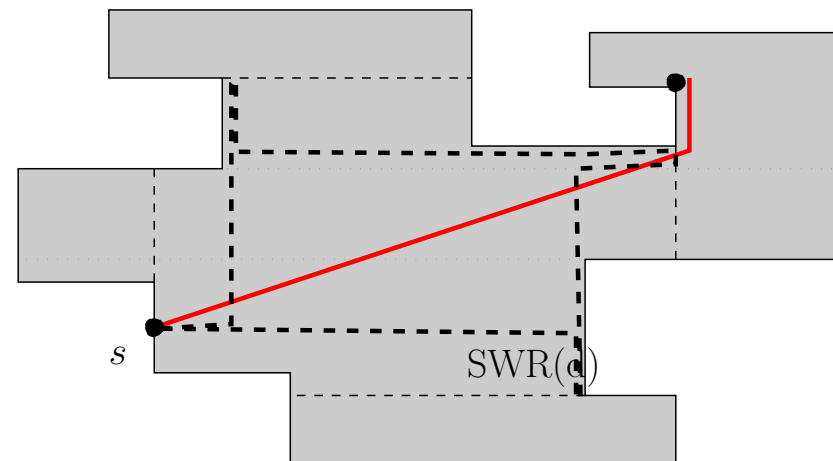
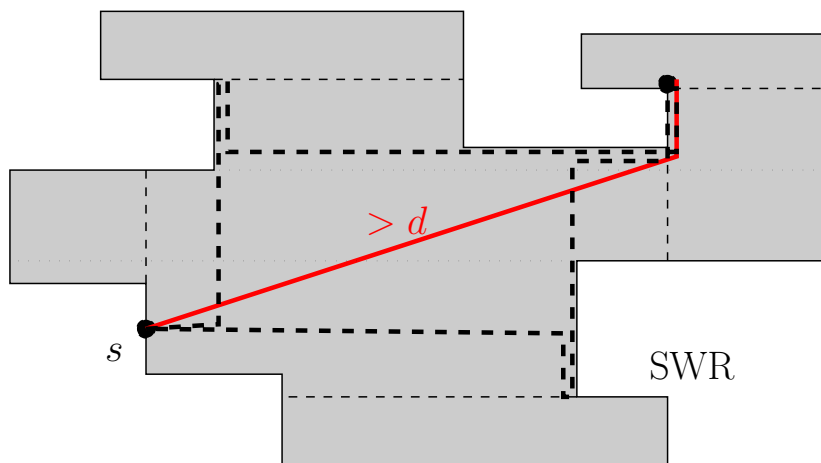
Rep.: L_1 -opt./ $\sqrt{2}$ -competitive! Theorem

- L_1 -paths, combination is also L_1 -optimal!
- L_1 -optimal path between any two points!
- Euclidean shortest path in between
- Triangle! Situation! Blackboard! $\sqrt{2}$ -Umweg maximal!



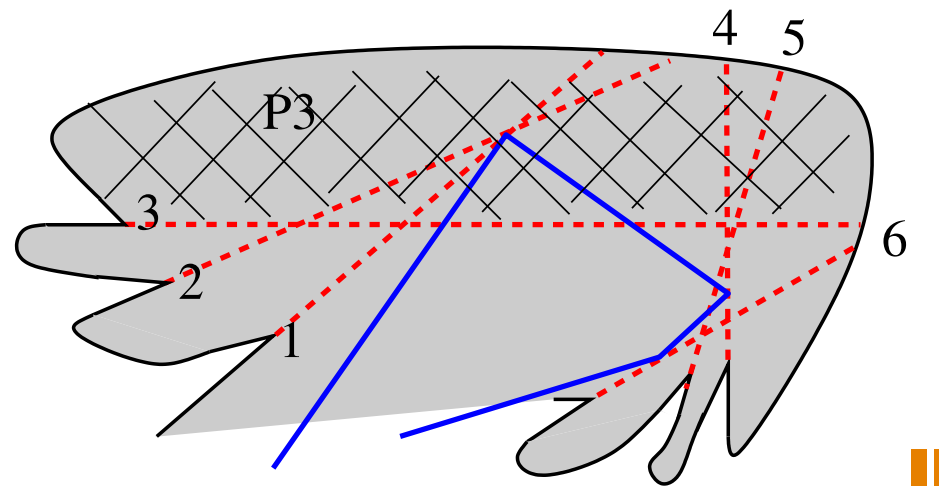
Rep.: L_1 -opt./ $\sqrt{2}$ -competitive! Theorem

- $\sqrt{2}$ -competitive
- Depth restricted!
- $\text{Expl}_{\text{ONL}}(d) \leq \sqrt{2} \text{Expl}_{\text{OPT}}(d)$
- **Theorem:** $8\sqrt{2}$ -Approximation

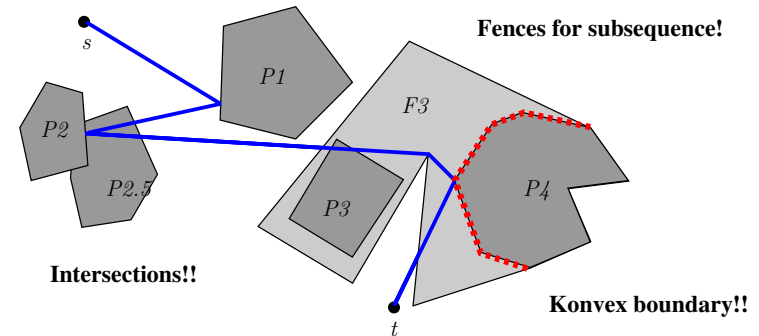
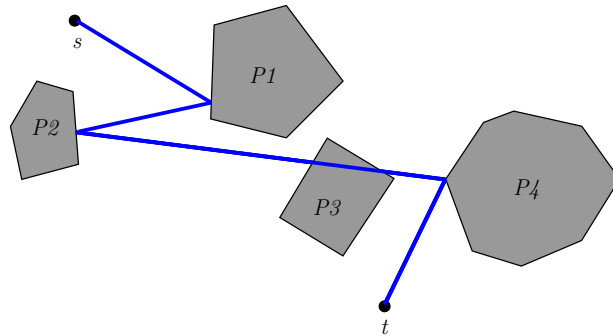


Rep.: SWR (General case): Offline!

- Corner problem!!
- Sequence of essential cuts, successive cuts
- Not visited by order along boundary.
- But the corresponding P_{c_i} !!!!



Rep.: Touring a sequence of polygons (TPP)

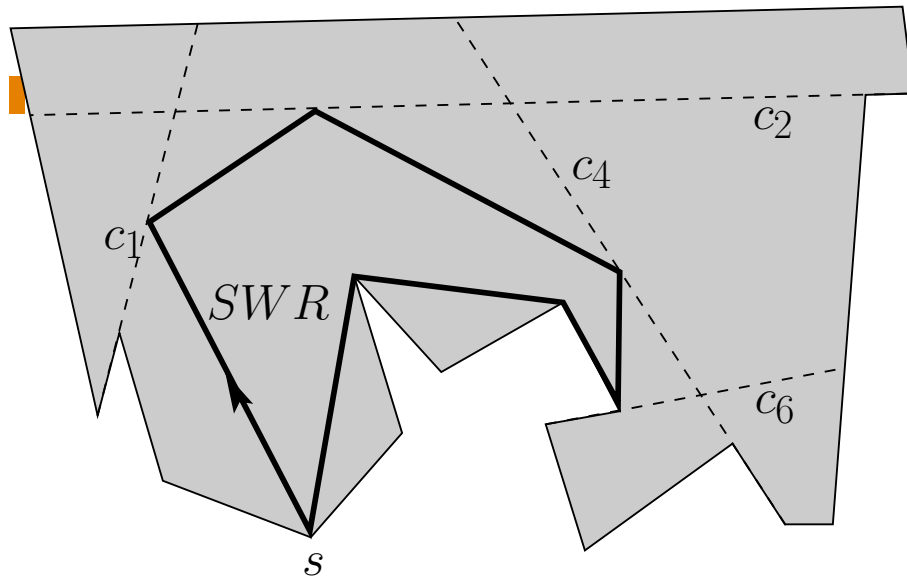


- Simple version:
- $O(nk \log \frac{n}{k})$
- Build(Query): $O(nk \log \frac{n}{k})$
- Compl.: $O(n)$
- Query (fixed s): $O(k \log \frac{n}{k})$

- General version:
- Fences, convex boundary, etc.
- $O(nk^2 \log n)$
- Build(Query): $O(nk^2 \log n)$
- Compl.: $O(nk)$
- Query (fixed s): $O(k \log n + m)$

Results from: Dror, Efrat, Lubiw, Mitchell 2003!!

Application: SWR



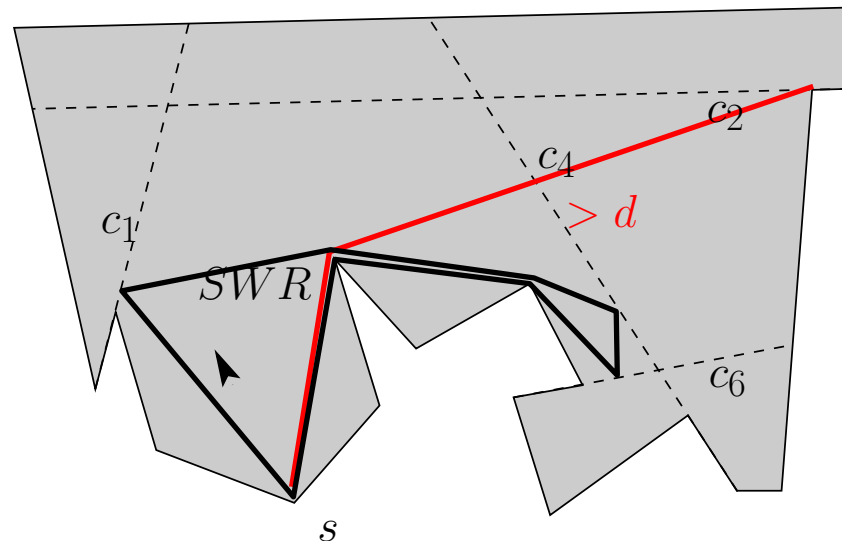
Essential *parts*! Use the order along the boundary!

One common fence, intersections!
Start and target identical!

- $O(n^4)$ '91
- $O(n^4)$ Tan et al. '99
- $O(n^3 \log n)$ by this result!
- **Theorem**

Rep.: Application: General simple polygons **Offline**

- Compute optimal exploration tour
- Agent with vision, start s at the boundary
- Depth restriction: Ignore cuts with distance $> d$
- $\text{Expl}(d) = \text{Expl}_{\text{OPT}}(d)$
- **Theorem:** δ Approximation, Online??



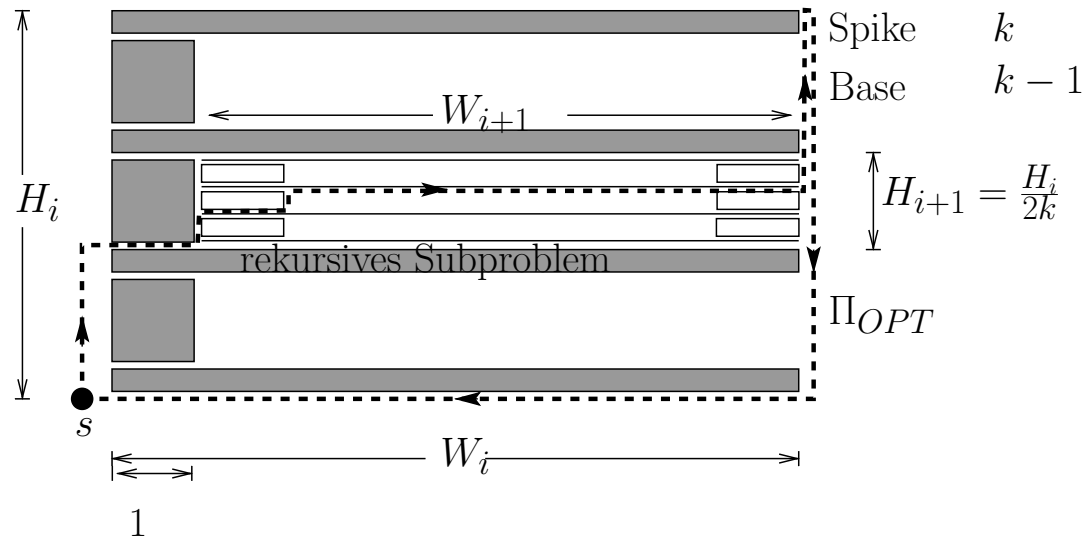
Rep.: Polygons with holes

There is no constant online approximation of the optimal search ratio

Theorem Let A be an online strategy for the exploration of a polygon with n obstacles (holes), we have: $|\Pi_A| \geq \sqrt{n}|\Pi_{OPT}|$

Proof: LB by examples!

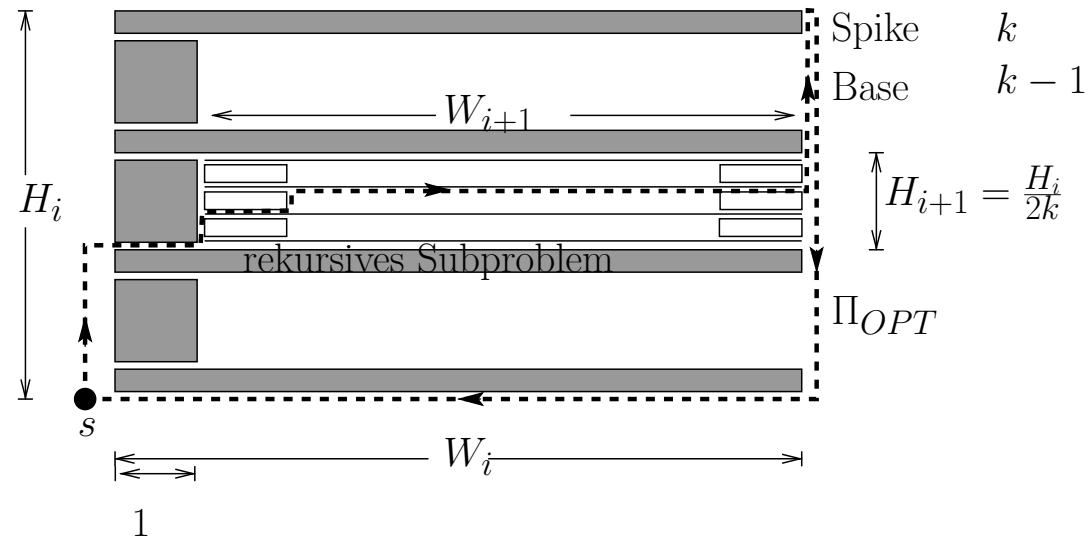
Rep.: Polygon with holes: $|\Pi_A| \geq \sqrt{n} |\Pi_{OPT}|$



- $W_1 = 2k$, $H_1 = k$, k spikes, $(k - 1)$ bases, $(2k - 1)k$ rectangles
- $H_i = \frac{H_1}{(2k)^{i-1}}$, $W_i = 2k - i + 1 \geq k$, $i = 1, \dots, k$
- Situation H_i : Online strategy does not know position of block H_{i+1}

- Left side: Look behind any block
- Right side: Move once upwards■
- Adversary: Find block after $\Omega(k)$ steps■
- Altogether: $\Omega(k \times k)$ for any Strategy ■

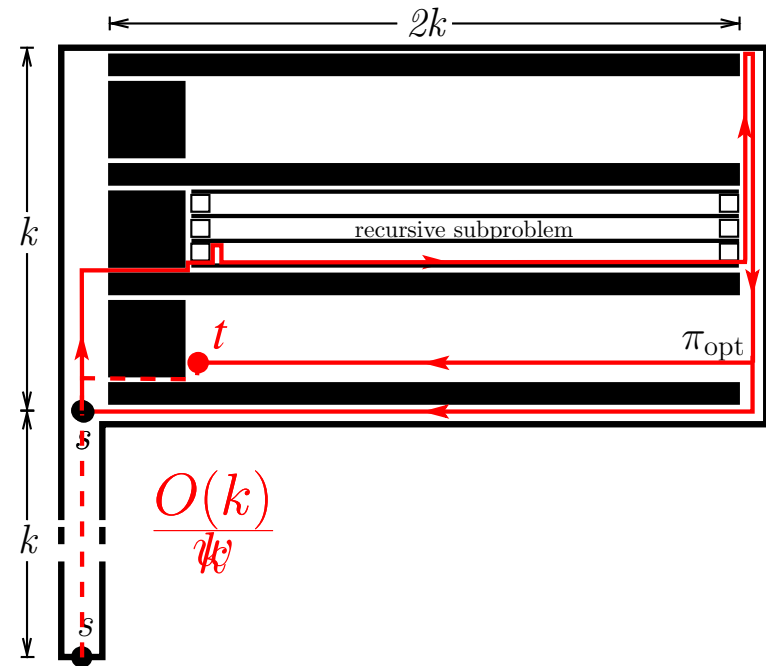
Rep.: Polygons with holes: $|\Pi_A| \geq \sqrt{n} |\Pi_{OPT}|$



- Optimale strategy: Move directly to the block
- Go on recursively, at the end move along any block
- $|\Pi_{OPT}| = W_1 + 2 \sum_{i=1}^k H_i \leq 6k$
- $k = \lfloor \sqrt{n} \rfloor$ gives the result

Rep.: Polygons with holes Corollary

- No $O(1)$ -competitive exploration for such environments ($\Omega(\sqrt{n})$)
- Optimal exploration has a bad Search Ratio
- Trick: Extension
- Then: Optimal exploration has Search Ratio $O(1)$
- Any online strategy has Search Ratio $\Omega(k)$

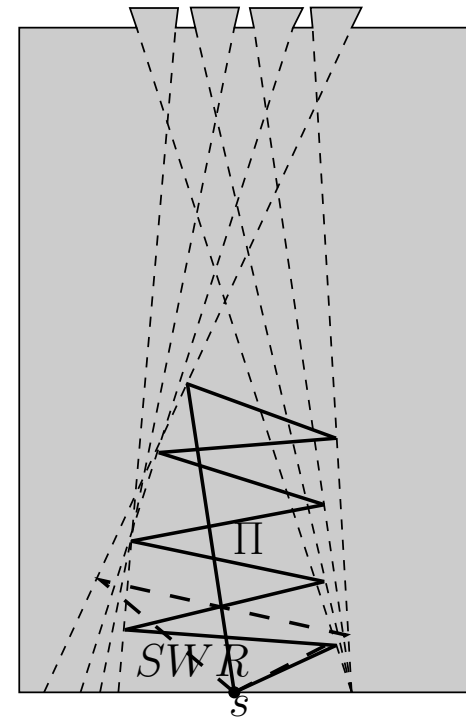


Rep.: Summary

- Connection between exploration and search:
- \exists constant-competitive, depth-restrictable exploration strategy
 $\Rightarrow \exists$ search strategy with mit competitiver Search Ratio
- \nexists constant-competitive exploration strategy,
but \exists 'extendable' lower bound
 $\Rightarrow \nexists$ search strategy with competitive Search Ratio

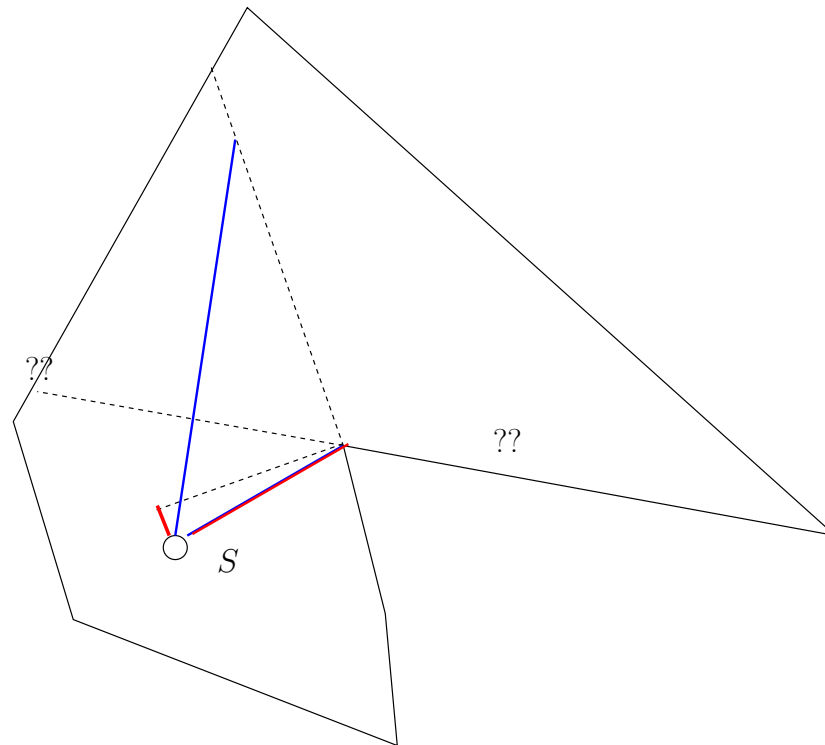
Online exploration general polygon

- Essential cuts by order along boundary
- Not competitive
- Subdivision into: Right and Left reflex vertices



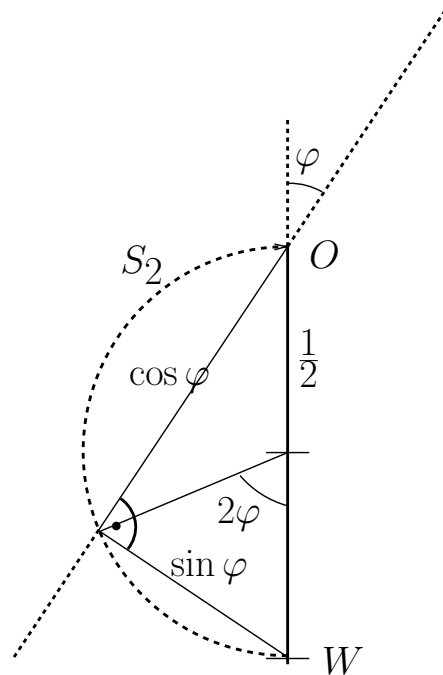
Online exploration general polygons

- Subtask: Explore a single corner
- Looking around a corner
- Simple strategies: Circular arcs!



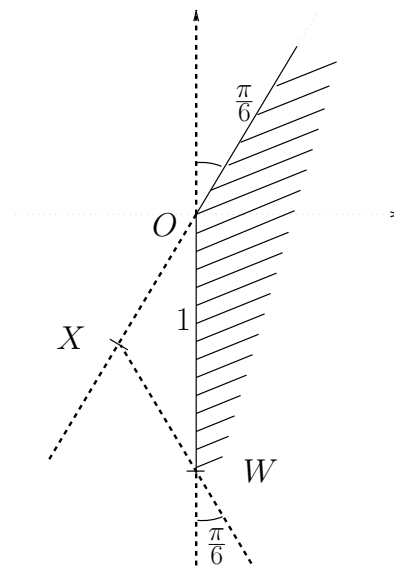
Looking around a corner, Theo.

- $H(\varphi) = \varphi$, monotonically increasing: Maximum for $\varphi = \frac{\pi}{2}$
- $G(\varphi) = \frac{\varphi}{\sin \varphi}$, $G'(\varphi) = \frac{\sin \varphi - \varphi \cos \varphi}{\sin^2 \varphi}$
- Optimize: $G'(\varphi) > 0$ für $\varphi \in (0, \pi/2]$, Max. for $\varphi = \frac{\pi}{2}$
- Ratio: $\frac{\pi}{2}$ because pathlength $\pi/2 \times 1$



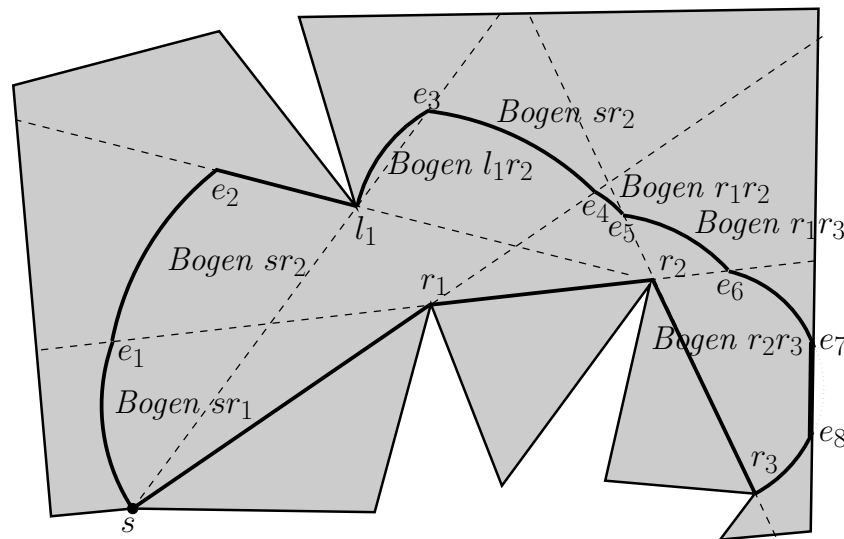
Looking around a corner: Simple lower bound!

- Special situation ■
- Strategy move left or right w.r.t. X ! ■
- In any case: Ratio $\frac{2}{3}\sqrt{3} = \frac{2}{\sqrt{3}}$, Blackboard! **Theo.** ■
- Optimal strategy: ratio 1.212... **Theo. (later)** ■



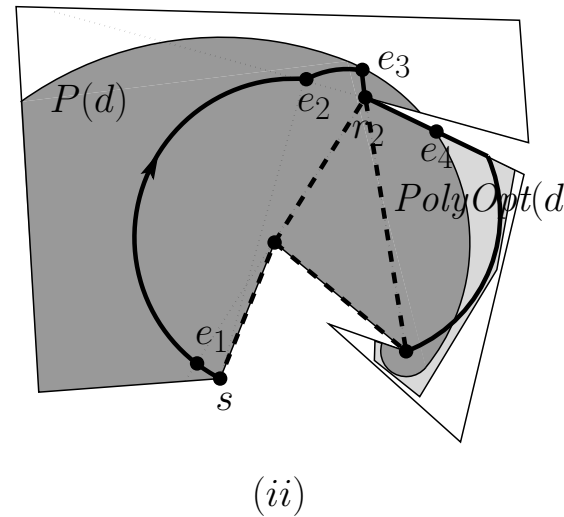
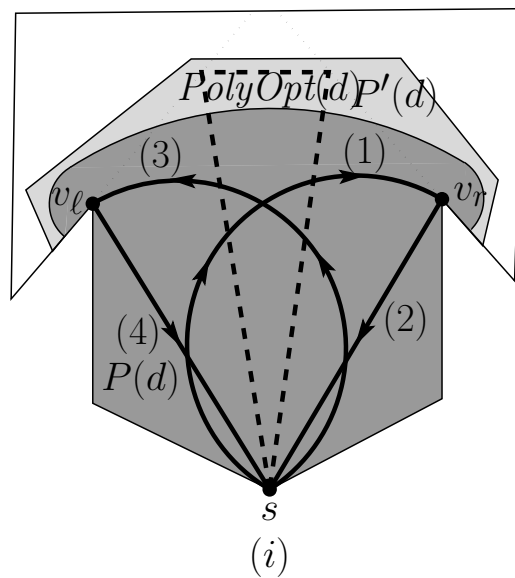
Online exploration general polygon

- Exploration by circular arcs
- Applet
- **Theorem** A simple polygon can be explored online within a competitive ratio of 26.5.
- Depth restrictable: $P(d)$, $\beta = 1$



PolyExplore: Eingeschränkte Exploration!

- Up to depth d , Beispiel! ■
- Ignore the cuts with distance $> d$, leave $P(d)$! Both! ■
- Analysis still holds! $\beta = 1$, $C_\beta = 26.5$ ■
- **Theorem** 26.5×8 Approximation of the optimal search path! ■



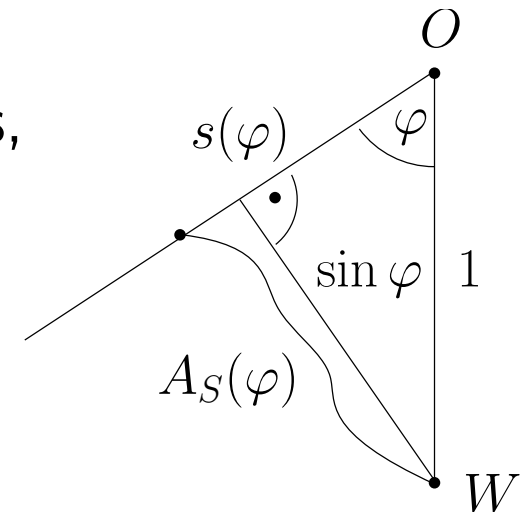
Look around a corner revisit: Optimal solution!

- Offline strategy: $d(\varphi)$ distance from W to extension $E(\varphi)$ at O

$$d(\varphi) = \begin{cases} \sin \varphi & : \varphi \in [0, \pi/2] \\ 1 & : \varphi \in [\pi/2, \pi] \end{cases}$$

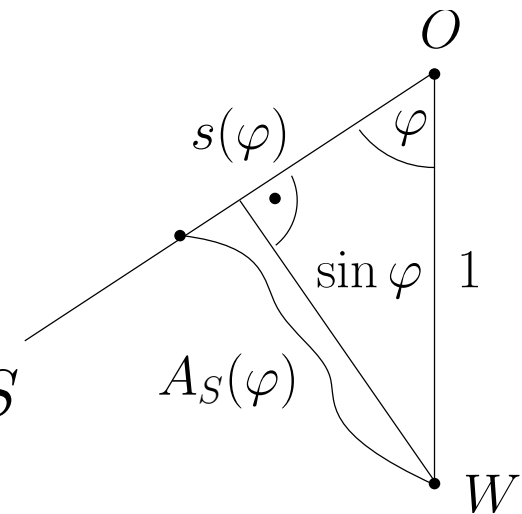
- Online strategy: $S(\varphi, s(\varphi))$ polar coordinates, $\varphi \in [0, \gamma]$, start $(0, -1)$, vertex $O = (0, 0)$.

1. Start at W : $s(0) = 1$
2. s continuous, no jumps, differentiable in $(0, \gamma)$
3. $s(\gamma) \neq 0 \Rightarrow \gamma = \pi$
4. $s'(0) < \infty$ (makes sense)



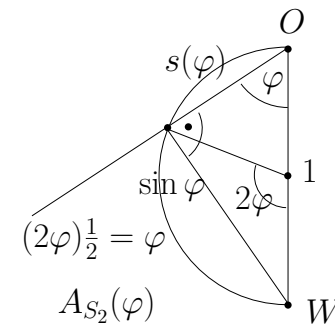
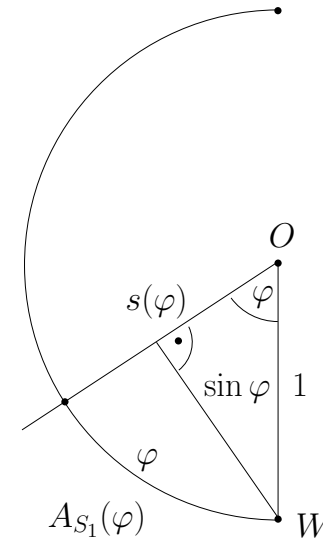
Look around a corner revisit: Optimal solution!

- Online strategy: $S(\varphi, s(\varphi))$ polar coordinates
- Path length: $A_S(\varphi)$ by $s(\varphi)$
- $A_S(\varphi) := \int_0^\varphi \sqrt{s'(t)^2 + s(t)^2} dt$
- Ratio: $R_S(\varphi) := \frac{A_S(\varphi)}{d(\varphi)}$
- S is c -competitive if:
 $R_S(\varphi) \leq c$ for all $\varphi \in [0, \gamma]$
- $c_s := \sup_{\varphi \in [0, \gamma]} R_S(\varphi)$ competitive ratio of S
- **Lemma:** S competitive $\iff |s'(0)| < \infty$
- Proof: Blackboard!
- Almost any strategy is successful!



Examples!

- $S_1(\varphi)$ half-circle with radius 1 around O
- $S_2(\varphi)$ half-circle with radius $\frac{1}{2}$ around midpoint of OW
- $R_{S_1}(\varphi) = \begin{cases} \frac{\varphi}{\sin \varphi} & : \varphi \in [0, \pi/2] \\ \varphi & : \varphi \in [\pi/2, \pi] \end{cases}$
- $c_{S_1} = R_{S_1}(\pi) = \pi = 3.14\dots$
- $R_{S_2}(\varphi) = \frac{\varphi}{\sin \varphi}, \varphi \in [0, \pi/2]$
- $c_{S_2} = R_{S_2}(\pi/2) = \pi/2 = 1.5708\dots$
- Upper bound: 1.5708, Lower Bound: 1.1547



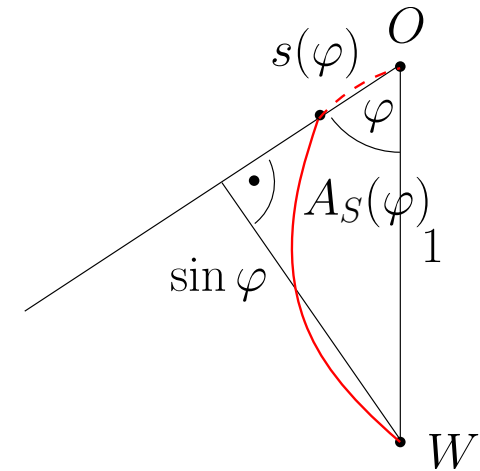
Strategy: Design

- Reasonable strategy: Window Shopper, Street Searching, Equality approach

1. Arrive at O with angle $\gamma \leq \pi/2$

2. $R_{S_2}(\varphi) = \frac{A_S(\varphi)}{\sin \varphi} = c_{\text{opt}}$ for all $\varphi \in [0, \gamma]$

- Design by requirements
- Proof of optimality



Strategy: Design

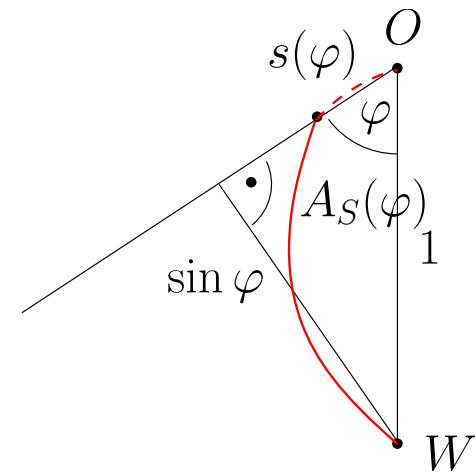
- $R_S(\varphi) = c_{\text{opt}}$ for all $\varphi \in [0, \gamma]$

-

$$c_{\text{opt}} = \frac{A_S(\varphi)}{\sin \varphi} = \frac{\int_0^\varphi \sqrt{s'(t)^2 + s(t)^2} dt}{\sin \varphi} \Leftrightarrow$$

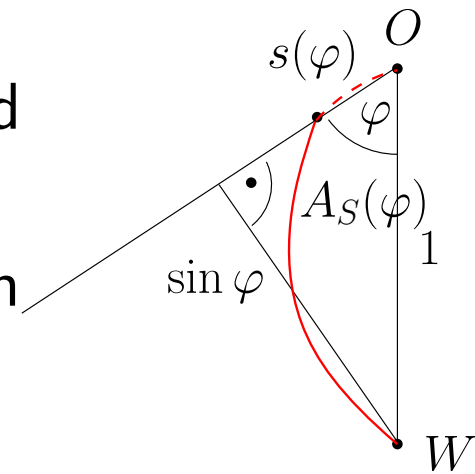
$$c \cdot \cos \varphi = (c \sin \varphi)' = \sqrt{s'(\varphi)^2 + s(\varphi)^2}$$

- $s(0) = 1, s(\gamma) = 0, s(\varphi) > 0$ for $\varphi \in (0, \gamma)$
- Solution: $s'(\varphi) = \pm \sqrt{c_{\text{opt}}^2 \cdot \cos^2 \varphi - s^2(\varphi)}$
- $s'(\varphi) = -\sqrt{c_{\text{opt}}^2 \cdot \cos^2 \varphi - s^2(\varphi)}$, decreasing radius!
- Trick: $s(\varphi) := c_{\text{opt}} r(\varphi)$ then
differential equation: $r'(\varphi) = -\sqrt{\cos^2 \varphi - r^2(\varphi)}$



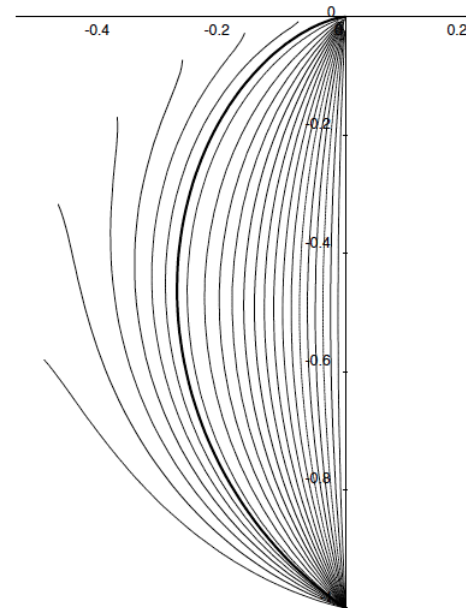
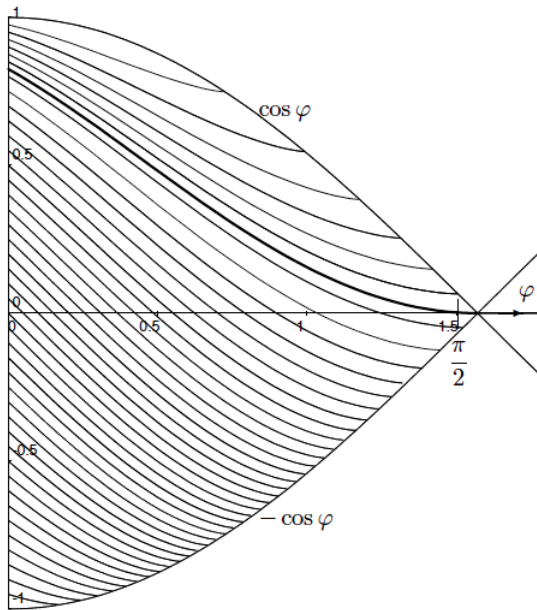
Strategy: Design

- $s(\varphi) := c \cdot r(\varphi)$
- $s(0) = 1, s(\gamma) = 0, s(\varphi) > 0$ for $\varphi \in (0, \gamma)$
- Ordinary differential equation:
 (1) $r'(\varphi) = -\sqrt{\cos^2 \varphi - r^2(\varphi)}$
- $r(0) = \frac{1}{c}, r(\gamma) = 0, r(\varphi) > 0$ for $\varphi \in (0, \gamma)$ and $r(\varphi) \leq \cos \varphi$
- Find minimum c such that (1) has solution with above properties
- Transformation (1) into:
 $w'(x) = (w^2(x) + 1)(1 - w(x) \cot x)$ no closed form solution!
- Only numerical solutions!



Strategy: Design

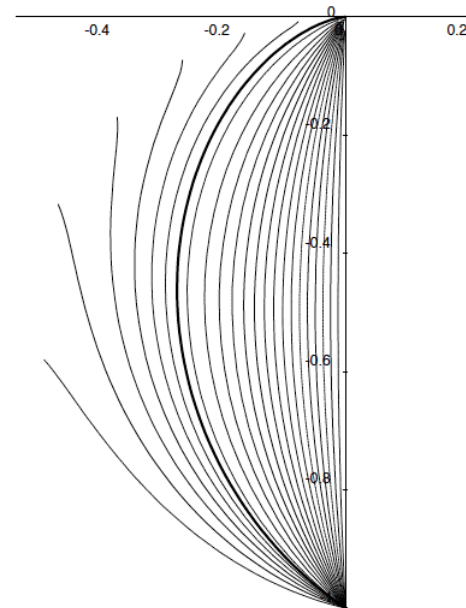
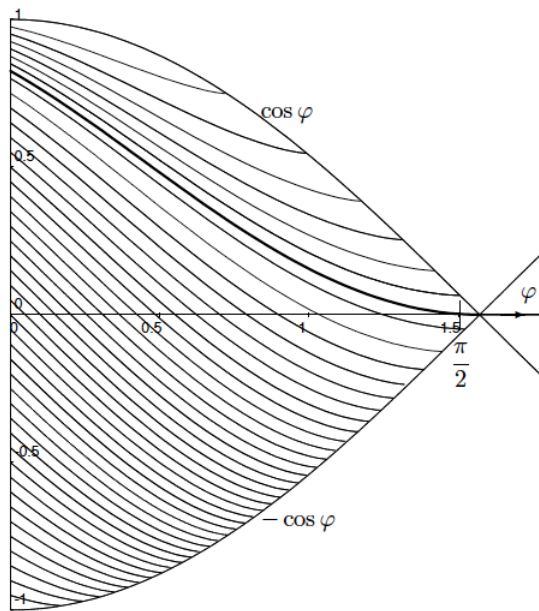
Ordinary differential equation: $r'(\varphi) = -\sqrt{\cos^2 \varphi - r^2(\varphi)}$
 $r(0) = \frac{1}{c}$, $r(\gamma) = 0$, $r(\varphi) > 0$ for $\varphi \in (0, \gamma)$ and $r(\varphi) \leq \cos \varphi$



Starting values $r(0) = \frac{1}{c}$, unique numerical solution! $r(\pi/2) = 0$!
Gives smallest c !

Strategy: Design

Lemma: For $r'(\varphi) = -\sqrt{\cos^2 \varphi - r^2(\varphi)}$ there is a unique solution $r(\varphi)$ satisfying $r(\pi/2) = 0$ and $r(\varphi) > 0$ for $\varphi \in [0, \pi/2)$.



Starting values $r(0) = \frac{1}{c}$, unique numerical solution! $r(\pi/2) = 0$
 Strategy $s(\varphi) = c \cdot r(\varphi)$, $c = 1.21218\dots$

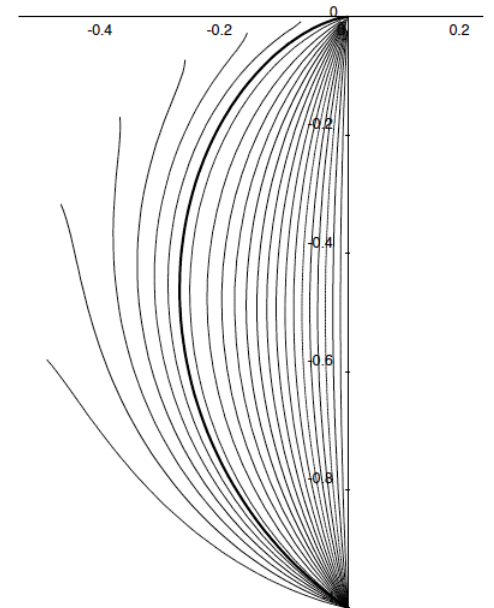
Strategy: Optimality

Theorem: The strategy S with $s(\varphi) = c \cdot r(\varphi)$ with $c = 1.21218\dots$ is the optimal strategy for looking around a corner

- Helping Lemma: Strategy S is convex!

Curvature formula: $\kappa := \frac{s^2 + 2(s')^2 - ss''}{((s')^2 + s^2)^{\frac{3}{2}}} > 0$

- Let T be arbitrary corner strategy. $s'_T(0) < \infty$
- Case I: $s'_T(0) \leq s'(0) < 0$
- $s'(0) = -\sqrt{c^2 \cos(0) - s(0)^2} = -\sqrt{c^2 - 1}$
- $\Rightarrow c_T \geq \sqrt{s'_T(0)^2 + 1} \geq \sqrt{s'(0)^2 + 1} = c$
- $\Rightarrow T$ worse than S



Strategy: Optimality

Theorem: The strategy S with $s(\varphi) = c \cdot r(\varphi)$ with $c = 1.21218\dots$ is the optimal strategy for looking around a corner

- Case II: $s'_T(0) > s'(0)$

- $\Rightarrow \exists \psi$ such that $s_T(\varphi) > s(\varphi) \forall \varphi \in [0, \psi]$

- Subcase I: $s_T(\psi) = s(\psi)$ for $\psi \leq \pi/2$:

$$c_S := \frac{A_S(\psi)}{\sin \psi} < \frac{A_T(\psi)}{\sin \psi} \leq c_T$$

- Subcase II: $s_T(\varphi) > s(\varphi)$ for $\varphi \in (0, \pi/2]$,

$$\psi > \pi/2$$

$$c_S := \frac{A_S(\pi/2)}{\sin \pi/2} < \frac{A_T(\pi/2) + s_T(\pi/2)}{\sin \pi/2} \leq c_T$$

- $\Rightarrow T$ worse than S

