

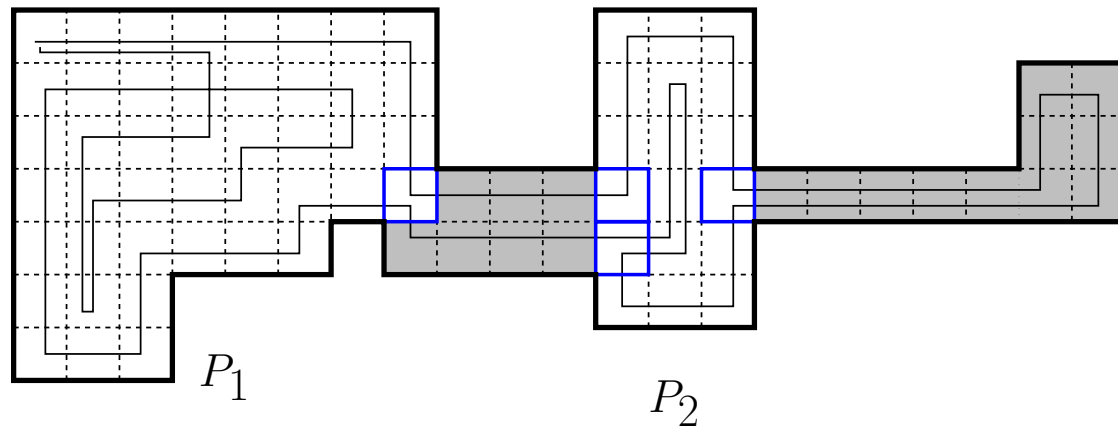
Online Motion Planning MA-INF 1314

General Grids

Elmar Langetepe
University of Bonn

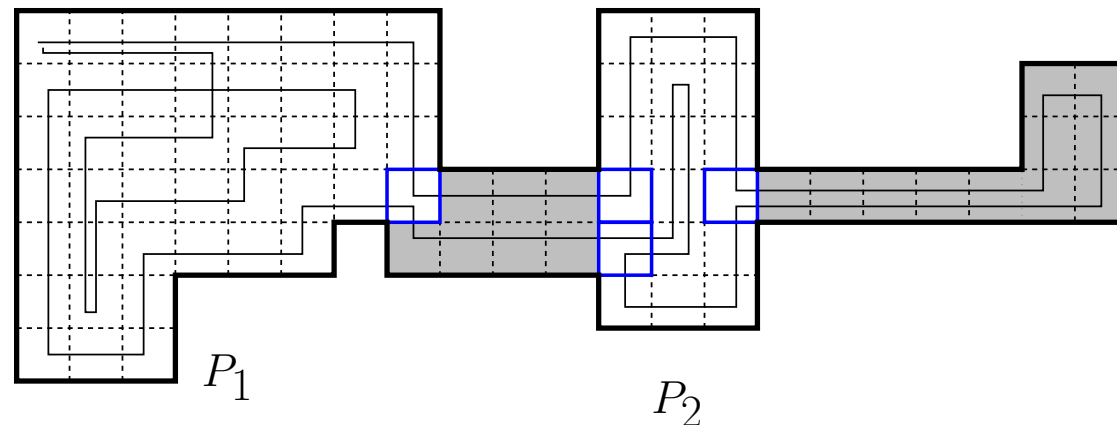
Repetition!

- SmartDFS: DFS Return path and component opt.■
- SmartDFS: Shortest return path■
- Wavefront Algorithmu (Lee): $O(n)$, n cells■
- Comp. Factor: $S(P) \leq \frac{4}{3} C(P) - 2$ (Untere Schranke $\frac{7}{6}$)■
- Observation: Optimally in narrow passages!■



Repetition

- Analyse polygons P_i , $i = 1, \dots, k$ ■
- Induction over split-cells ■
- Induction-Base: No split-cell in layer 1. ■
- **Lemma** $E(P) \leq \frac{2}{3}C(P) + 6$ ■ Backward analysis ■
- **Lemma** $S(P) \leq C(P) + \frac{1}{2}E(P) - 5$ ■ Two steps less by Offsetlemma! ■
- Kombination gives Induction-Base! ■

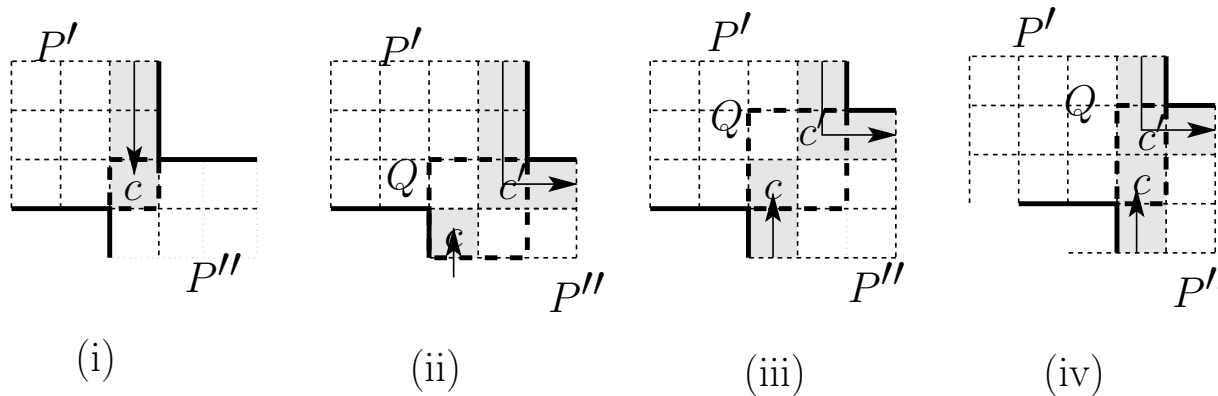


Induction-Step: Fall (i): $S(P_i) \leq \frac{4}{3}C(P_i) - 2$

- $S(P_i) = S(P') + S(P'')$ (Gate) $C(P_i) = C(P') + C(P'') - 1$
- IH. for P' and P''

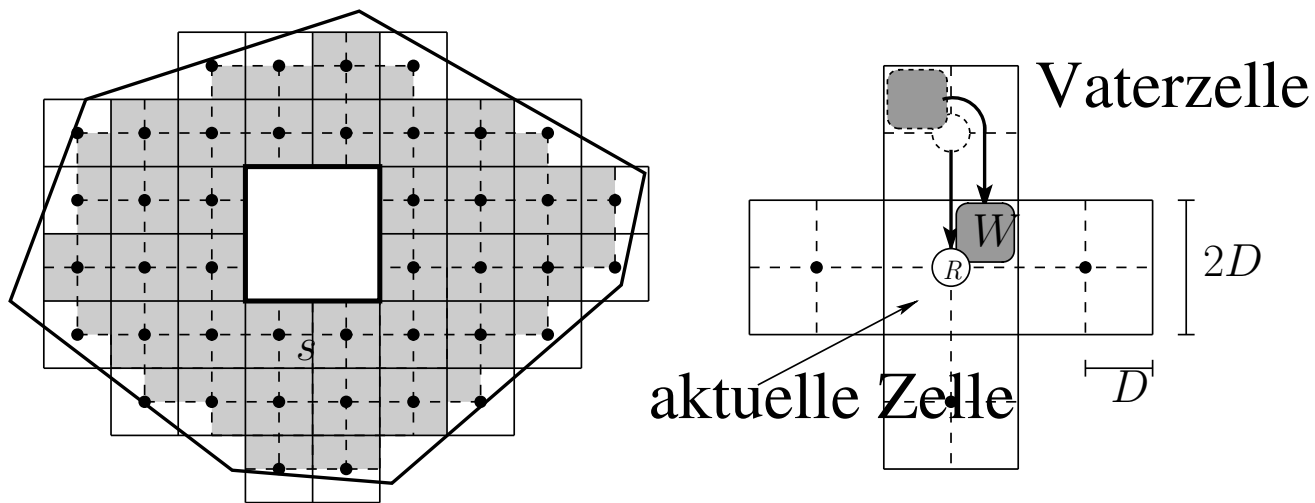
$$S(P_i) = S(P') + S(P'') \leq \frac{4}{3}C(P') - 2 + \frac{4}{3}C(P'') - 2$$

$$\leq \frac{4}{3}C(P_i) + \frac{4}{3} - 4 < \frac{4}{3}C(P_i) - 2$$



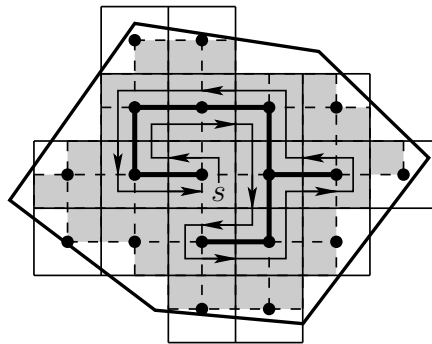
General gridpolygons

- Change the modell, due to the analysis
- $2D$ cells with center, sub-cells
- See adjacent $2D$ Zellen
- Tool W of size D

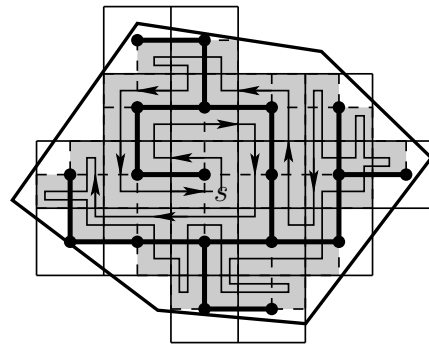


Spanning Trees

- Online with DFS with a Spanning Tree of 2D vertices
- Move the tool along the tree
- Left-Hand-Rule along the tree
- Variants 2D cell totally free for the edge/or not!
- Any cell only once or more than once



Nur unbelegte 2D Zellen



Belegt aber begehbare 2D Zellen

2D Spiral STC: 2DSPSTC(*parent*, *current*)

Mark *current* as explored

while *current* has unexplored neighbour **do**

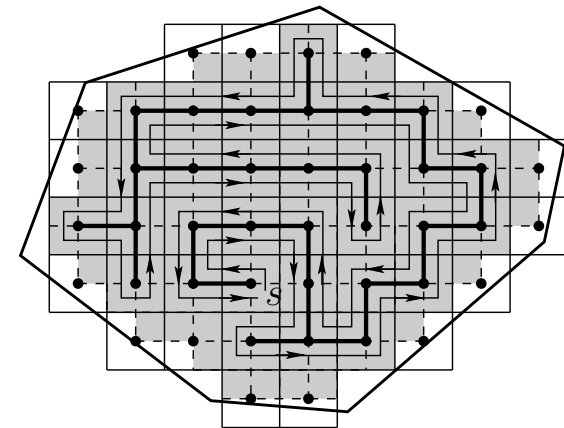
- Search from *parent* in ccw order
neighbour *free* non-explored/free
- Span.Tree edge *current* zu *free*.
- Move tool L-H-R along
Span.Tree edge to
first sub-cell of *free*
- 2DSPSTC(*current*, *free*)

end while

if *current* \neq *s* **then**

- From *current* by L-H-R along
Span.Tree to subcell of *parent*

end if

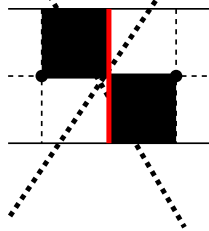


Nur freie 2D Zellen

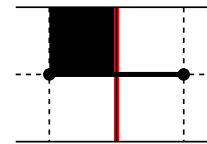
Partially blocked 2D cells

- Spanning Tree, edge is free/not the full cell
- Reachable D sub-cells?
- Different types
- **Definition:** double-sided edge, one-sided edge

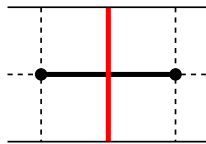
Keine Kante



Einseitige Kante



Doppelseitige Kante



Spezialfall

2D Spiral STC: 2DSPSTC(*parent*, *current*)

Mark *current* as explored

while *current* has unexplored neighbour **do**

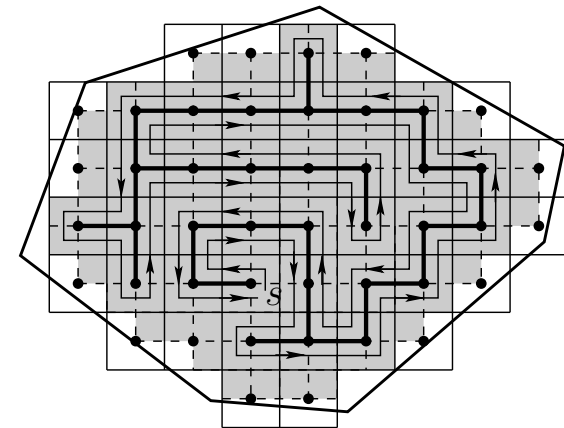
- Search from *parent* in ccw order
neighbour *free* non-explored/*free*
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- 2DSPSTC(*current*, *free*)

end while

if *current* \neq *s* **then**

- From *current* by L-H-R along
Span.Tree to subcell of *parent*

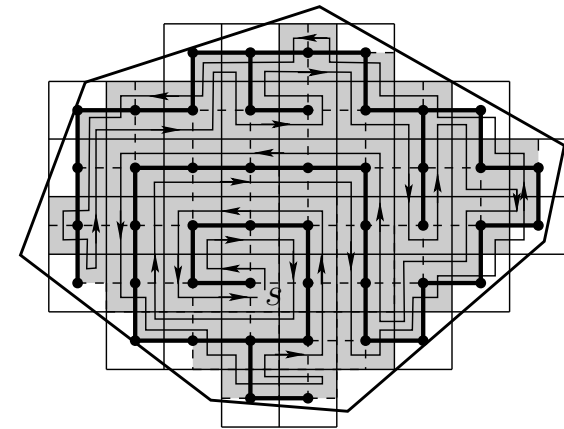
end if



Nur freie 2D Zellen

Spiral STC: $SPSTC(parent, current)$

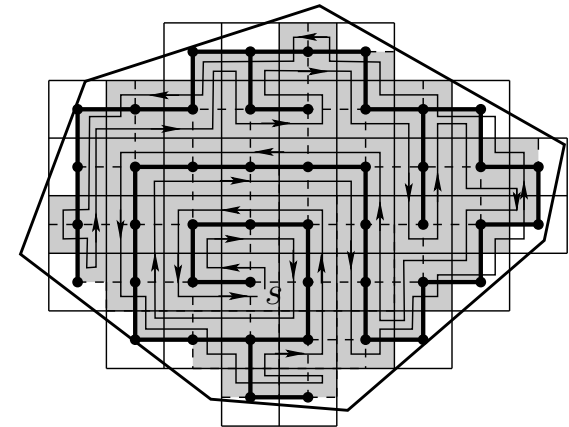
- Search from *parent* in ccw order neighbour *free*/non-explored, s.th. spanning tree edge can be build (might be single-sided)
- Search from *parent* in ccw order neighbour *free* non-explored/*free*



Falls Knoten erreichbar

Spiral STC: $SPSTC(\textit{parent}, \textit{current})$

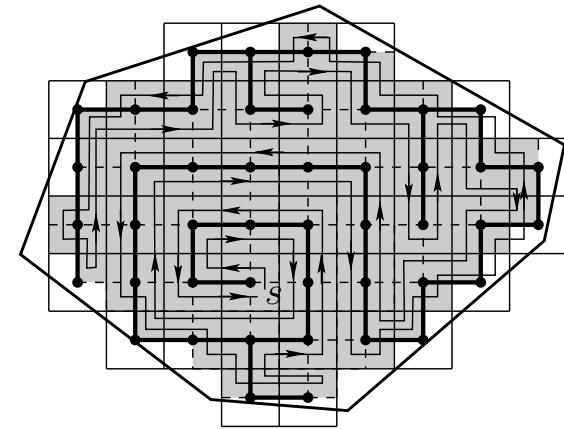
- Move tool along the spanning tree edge to the first reachable sub-cell of *free*. Left-Hand-Rule for double-sided edges. Avoid obstacles of single-sided edges. Tool might change to the left of the spanning tree edge.
- Move tool L-H-R along Span. Tree edge to first sub-cell of *free*



Falls Knoten erreichbar

Spiral STC: $SPSTC(\textit{parent}, \textit{current})$

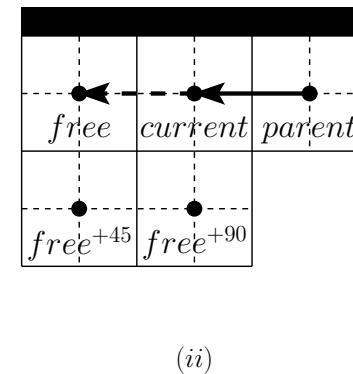
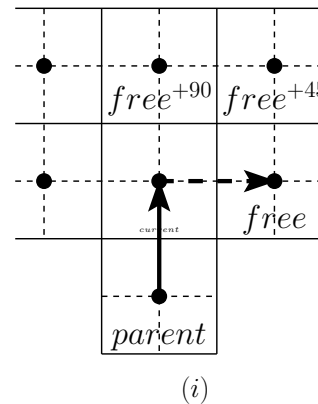
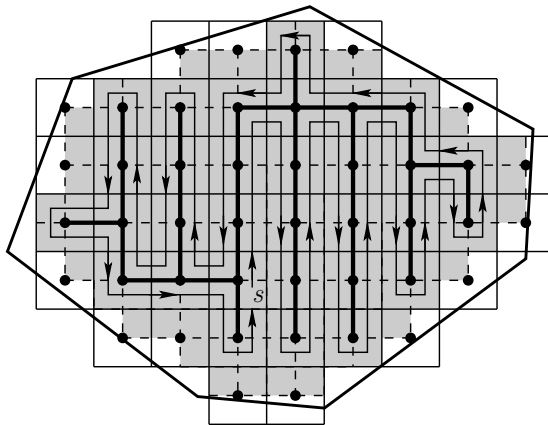
- Move tool along the spanning tree edge to the first reachable sub-cell of *free*. Left-Hand-Rule for double-sided edges. Avoid obstacles of single-sided edges. Tool might change to the left of the spanning tree edge.
- From *current* by L-H-R along Span.Tree to subcell of *parent*



Falls Knoten erreichbar

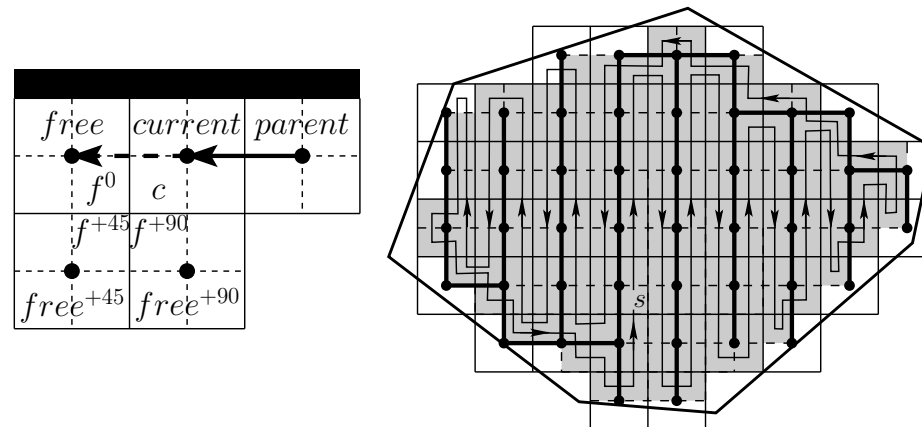
Less rotations for the tool

- Avoid spiral-like paths
- ● Move in columns
- Scan also diagonally adjacent 2D cells
- ScanSTC 2D Algorithm
- Also for the Backtracking step



Less rotations for the tool

- Avoid spiral-like paths
- ● Move in columns
- Also for the general case/path should exist
- Scan also diagonally adjacent 2D cells
- ScanSTC Algorithm
- Also for the Backtracking step



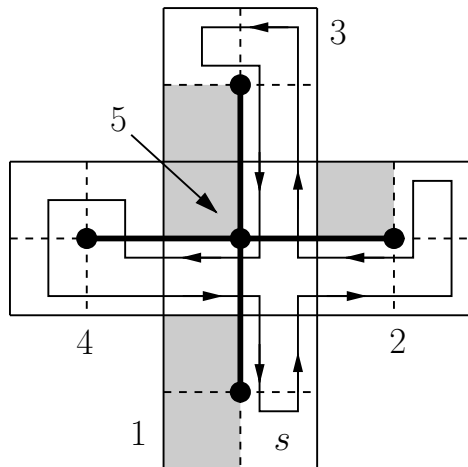
Analysis! Theorem

- General Spiral STC ■
- Number of steps for the tool ■
- As given by SmartDFS, C plus overhead ■
- D sub-cells, at the *boundary* K in total ■

P gridpolygon, C reachable sub-cells. K reachable sub-cells that are diagonally adjacent to a blocked sub-cell. P is explored by Spiral-STC or Scan-STC. Requires $O(C)$ space and $O(C)$ time. The number of steps for the tool is restricted to $S \leq C + K$. ■

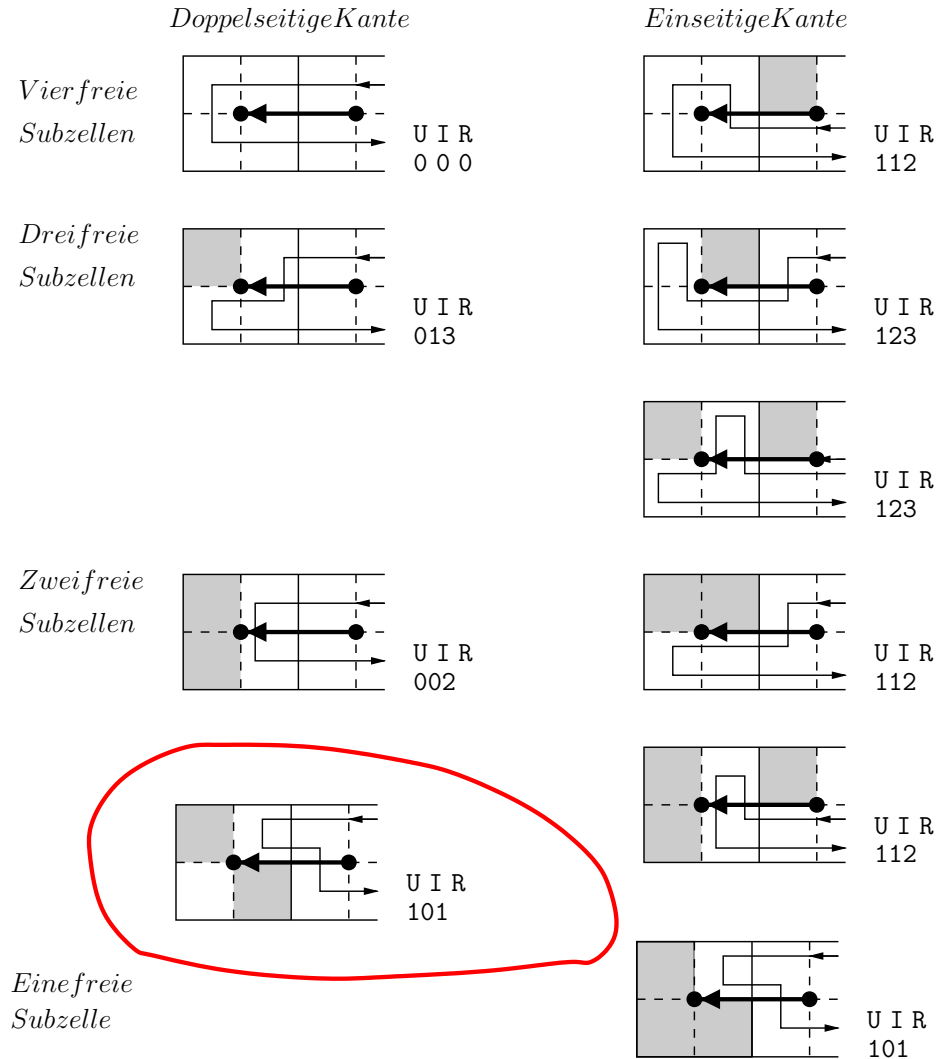
Number of steps! Example!

- Locally, count boundary sub-cells
- Local analysis, multiple visits, charge boundary sub-cells
- 2D Inner-cell/ Intra-cell
- Systematically: Boundary sub-cells charged by *Inner* plus *Intra*



Zelle	Übergr.	Intern	Gesamt	Randzellen
1	0	1	1	2
2	1	2	3	3
3	1	2	3	3
4	1	1	2	2
5	1	2	3	3

Number of steps! Theorem Systematically!

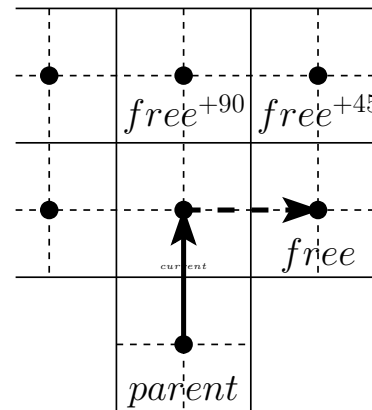
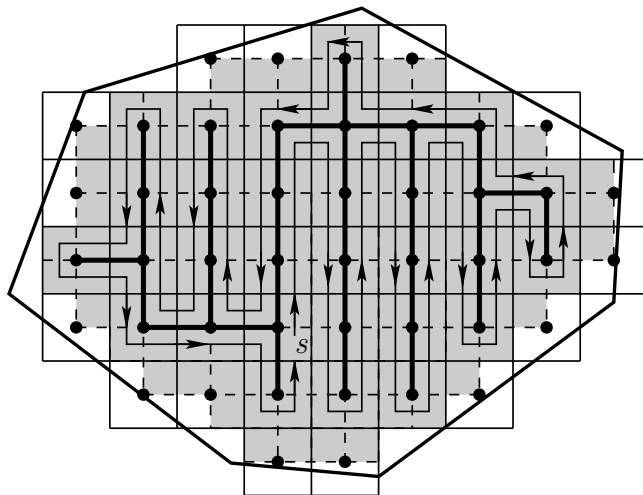


Running time and space required Theorem

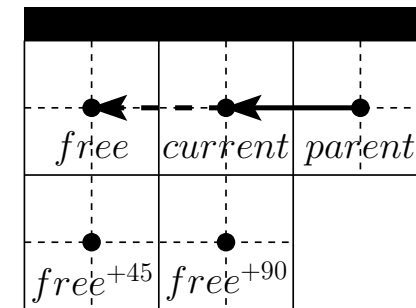
- $C + K$ steps but K is bounded by C ■
- ● Local decisions: $O(1)$ ■
- Running time and space $O(C)$ ■

Analysis of 2D-ScanSTC

- Give a Scan-Preference: I.e. Vertically
- Decide only locally (more information)
- How many *bad* horizontal edges?
- Optimal number! H_{opt}
- Compare with 2D-ScanSTC: Say H_{STC}



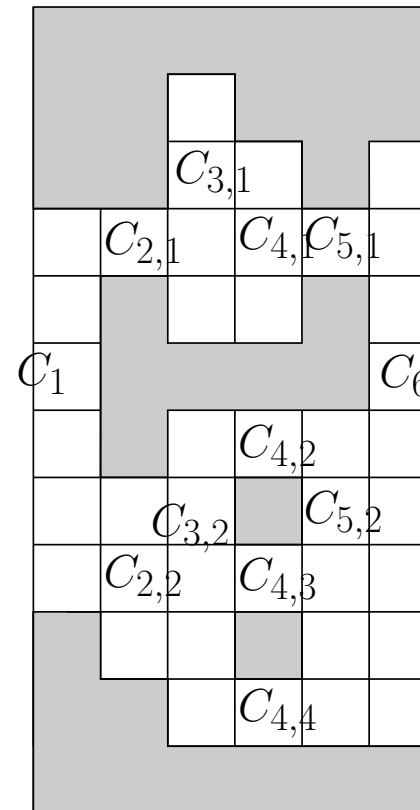
(i)



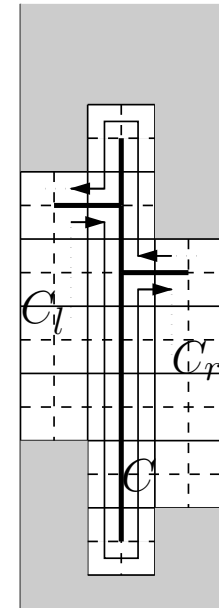
(ii)

Analysis of 2D-ScanSTC

- Columns connectivity
- From Left to Right X nach Y
- Sum up the Differences: Overall Z
- Connectivity changes



(i)



(ii)

Proof Sketch

- H_{Opt} optimal number of horizontal edges in the spanning tree. Z number of connectivity changes of P . 2D-Scan-STC requires

$$H_{STC} \leq H_{Opt} + Z + 1$$

horizontal edges in its spanning tree.

