Online Motion Planning MA-INF 1314
Graphexploration/Marker

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Repetition: CFS Algorithm Invariants Lemma

Execution CFS–Algorithm, properties hold:

i) Any incomplete vertex belongs to a tree in $\mathcal{T}$.

ii) There is always an incomplete vertex with $v \in V^*$ with $d_{G^*}(s, v) \leq r$, until $G^* \neq G$.

iii) For any chosen root vertex s_i: $d_{G^*}(s, s_i) \leq r$.

iv) After pruning T_i is fully explored by DFS. All trees $T \in \mathcal{T}$ have size $|T| \geq \frac{\alpha r}{4}$.

v) All trees $T \in \mathcal{T}$ are disjoint (w.r.t. edges)

Proof: i) and v) simply hold by construction

Rep Analysis Theorem/Corollary

CFS–Algorithm known depth r $(4+\frac{8}{\alpha})$–competitive/cost $\Theta(|E|+|V|/\alpha)$.

über Teilbäume T_R

- Subtree T_R, cost
- $K_1(T_R)$: path from s to s_i in G^*
- $K_2(T_R)$: DFS, $K_3(T_R)$: bDFS (Graph!)
- $\sum\limits_{T_R} K_3(T_R) \leq 2 \cdot |E|$ bDFS global
- $\sum\limits_{T_R} K_2(T_R) = \sum\limits_{T_R} 2 \cdot |T_R| \leq 2 \cdot |E|$, DFS, disjoint
- $\sum\limits_{T_R} K_1(T_R) \leq \sum\limits_{T_R} 2r \leq \frac{8}{\alpha} \sum\limits_{T_R} |T_R| \leq \frac{8}{\alpha}|E|$

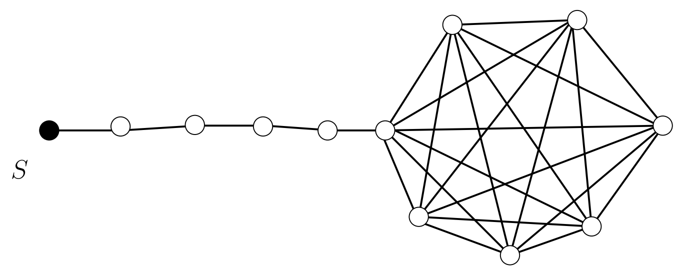
Rep: Graphexploration, unknown depth r

- Doubling-Heuristic: $O(|E| + \log r|V|)$ steps Schritte, **Corollary**
- Adjust prune/explore with current value $d_{G^*}(s, s_i)$
- prune$(T_i,\ s_i,\ \frac{\alpha d_{G^*}(s,s_i)}{4},\ \frac{9\alpha d_{G^*}(s,s_i)}{16}\ )$
- explore$(\ \mathcal{T},\ T_i,\ s_i,\ (1+\alpha)d_{G^*}(s, s_i)\ )$
- **Lemma iv)**: Rest of T_i fully explored by DFS, all $T \in \mathcal{T}$ have size $|T| \geq \frac{d_{G^*}(s,T)\alpha}{4}$
- **Theorem**/**Corollary** CFS–Algorithm unknown depth R is $(4 + \frac{8}{\alpha})$–competitive/has cost $\Theta(|E| + |V|/\alpha)$

Look-ahead $\alpha \cdot r$ necessary

Lower bound $\Omega(|E|^{1+\epsilon})$ Offline accumulator variant, if look-ahead is smaller than linear in r (constant).

- $2r$ is not sufficient: At least $2r+1$!
- With $2r+o(r)$ not efficient! (small-o notation!)
- Graph: path and clique, beyond linear
- Accumulator size $n+f(n)$: $\Omega\left(\frac{n^3}{f(n)}\right)$ Schritte!
- $|E| \in C \cdot n^2$, $f(n) = n^{1-\epsilon}$
- Conjecture: $r+o(r)$ is not sufficient for tether variant. Open!!



Look-ahead $\alpha \cdot r$ necessary

Lemma For the accumulator variant with accumulator size of $2r + d$ for constant d there are examples where $\Omega(|E|^{\frac{3}{2}})$ exploration steps are necessary.

Proof: Blackboard!

Note: It can still be competitive!

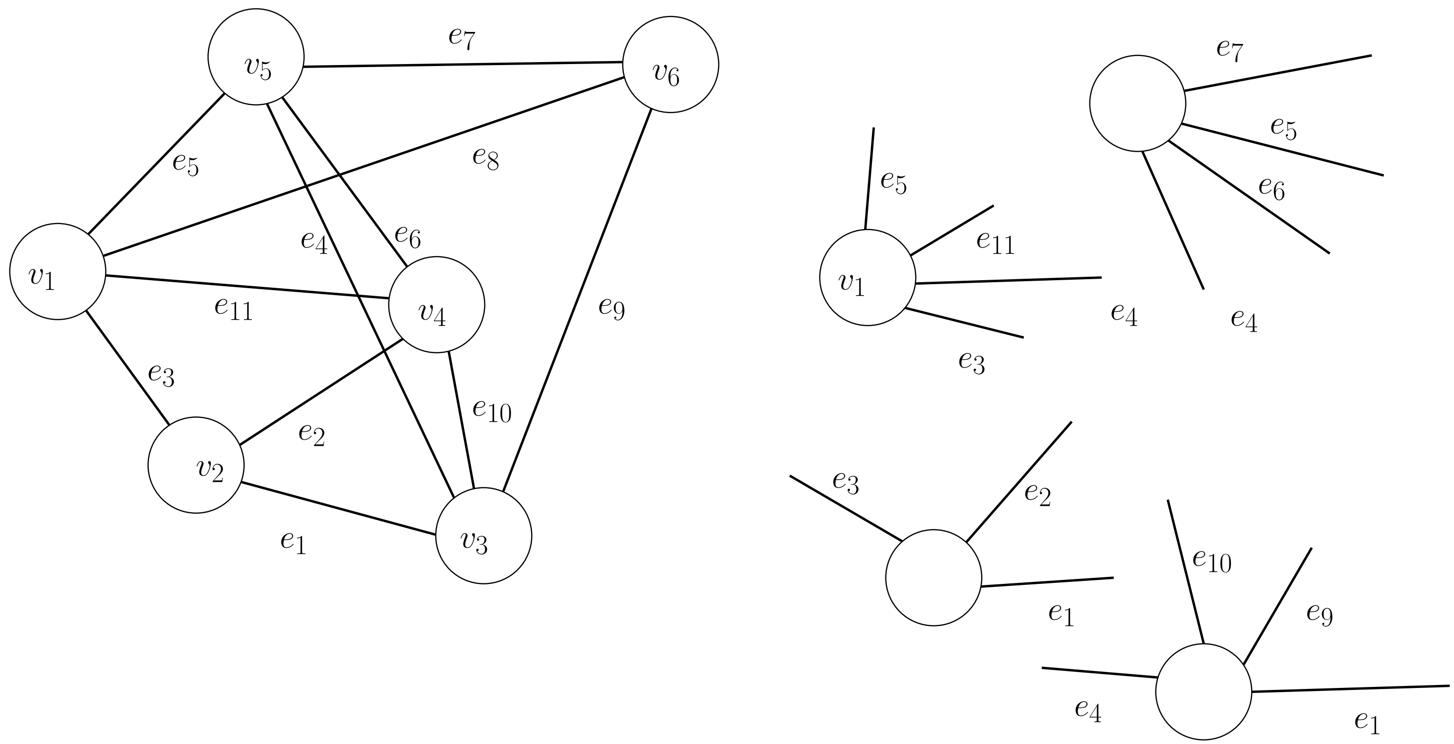
Offline cost?

- Mechanical cost/Computational cost
- Build the spanning trees
- Move along the shortest path
- Merge the trees
- DFS/bDFS
- Not all linear
- Exercise

Different model

- Vertices/Edges have been marked
- As a landmark
- Assume: This is not possible! How to distinguish?
- Vertices cannot be distinguished immediately!
- Local order of the edges is given
- May be not a planar embedding!
- Given: $G = (V, E, S)$, S cyclic orders!

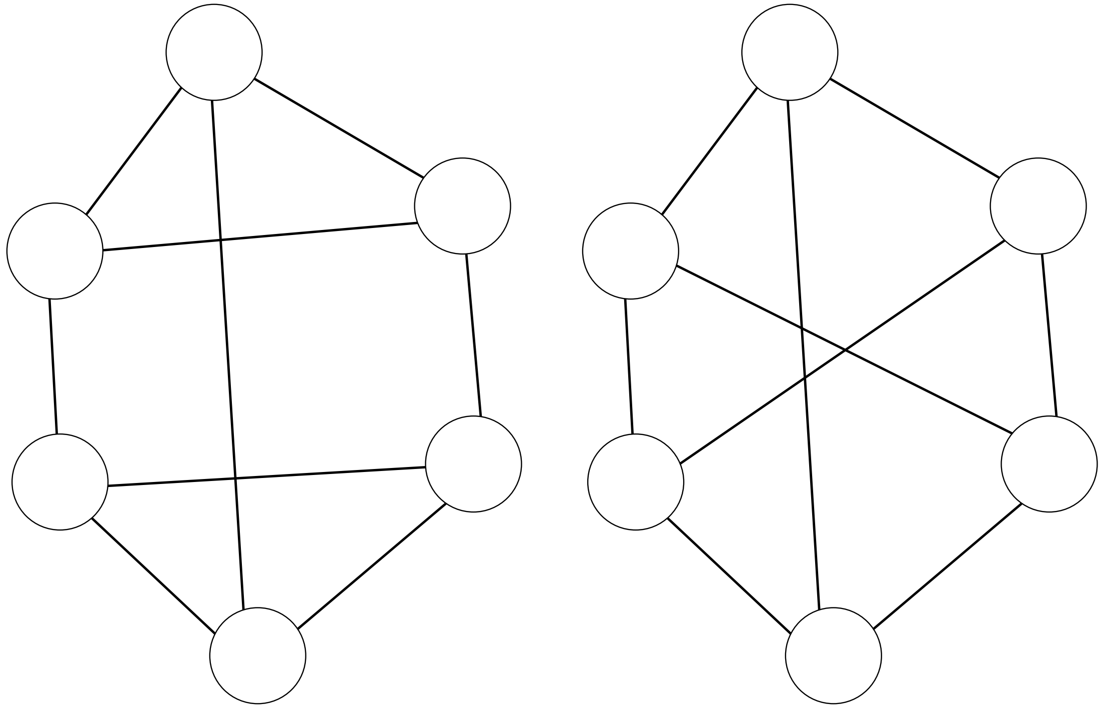
Different model, local order



From different vertices, permutation! Locally fixed

Mapping problem!

- Determine the graph (for navigation!)
- Store all given information
- Marker/pebble is necessary



One-Marker Algorithm (Dudek et al.)

- Maintain known graph S
- List L of adjacent unknown edges
- Choose edge $e \in L$ from some $b \in S$
- Visits vertex u
- Put pebble/marker at u
- Search in S from b for the pebble
- If marker was not found, add edge(b, u) and vertex u to S
- Insert the adjacent edges from u into L
- If marker has been found at known vertex $v = u$, try to search for the edge $e = (b, v)$ by the order from b
- For this: Place marker onto b, move to b and then in S back to $v = u$ along shortest path

- Check the outgoing edges for
- One will be the right one! Update S!
- Pseudocode! Exercise!

Analysis: One-Marker Algorithmus

- Mechanical cost: Number of steps!!
- Assumption: No loops!
- Set the marker $O(1)$
- Search for the marker: DFS on vertices $2|V_S|$
- Bring the marker back, move back: $2|V_S|$
- Do this for all possible edges: $O(|E| \times |V|)$

Analysis: One-Marker Algorithm

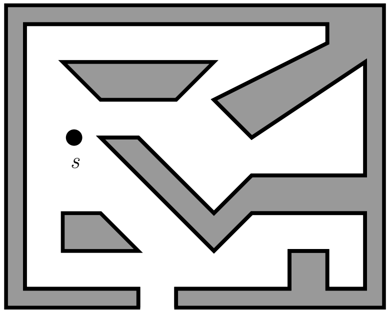
- Computational cost: Offline!
- Shortest path in graphs
- Dijkstra: $O(|E_S| + |V_S| \log |V_S|)$
- For any edge
- $O(|E|^2 + |E||V| \log |V|)$

Graph-exploration

- Labyrinths, grid-graphs, gridpolygons, general graphs
- Graph-exploration: DFS and LB of 2
- Gridpolygons: Simple/general
- SmartDFS $\frac{4}{3}$, LB $\frac{7}{6}$
- STC Alg. $|C| + |B|$
- Tether/Accumulator/Depth variants: $\Theta(|E| + |V|/\alpha)$
- Marker Algorithm
- Online TSP for planar graphs!

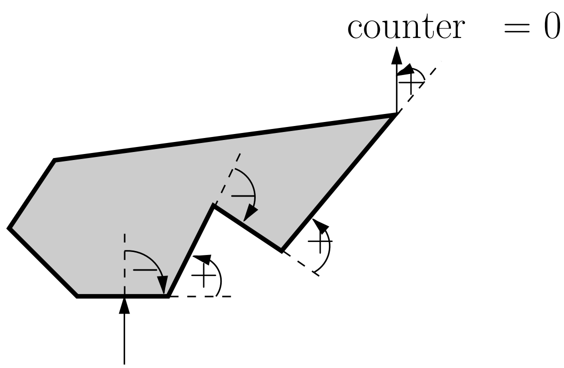
Kap. 2: Polygonal environments

- Set of disjoint simple polygons in the plane
- Boundary polygon
- Different tasks: Searching for a goal/escape from a labyrinth
- Different sensor models
- First: Touch sensor, precise odometry, escape from a labyrinth



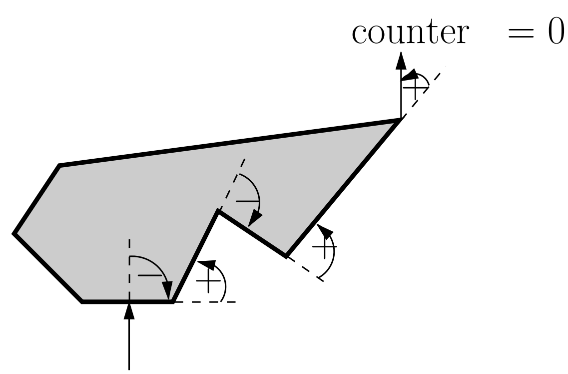
Escape from a labyrinth: Model

- Point-shaped agent
- Touch sensor
- Follow the wall
- Follow a direction (exact)
- Count rotational angles, in total
- No further memory



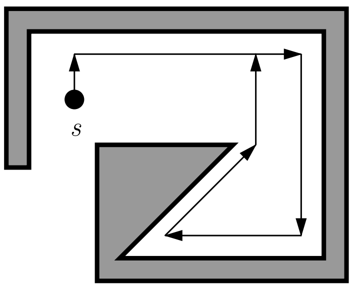
Pledge Algorithm

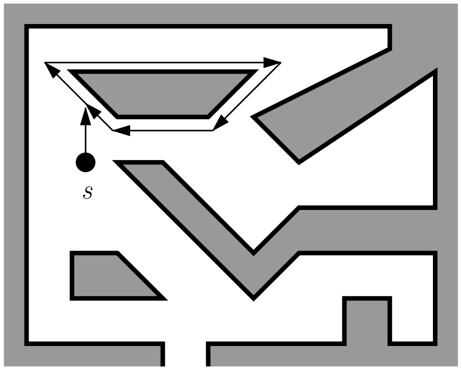
1. Choose angle φ, rotate agent heading in this direction.
2. Move into direction φ, until agent reaches the boundary.
3. Move right and keep in contact with the wall, Left-Hand.
4. Follow the wall by Left-Hand-Rule and sum up the rotational angles, until the overall rotational angle attains value zero, now GOTO (2).



Pledge Algorithm

- Angular counter $\bmod 2\pi = 0$, not sufficient
- Only Left-Hand-Rule not sufficient





Correctness, structural properties, non-negative counter

Lemma The angular counter of the Pledge Algorithm is never positive.

Proof:

- Zero at the beginning
- Zero, when the boundary is left
- Right turn after hitting the boundary $\Rightarrow$ negative
- Continuous change, zero $\Rightarrow$ movement is possible

Correctness, no-success, finite path repeated

Lemma If the agent does not leave the labyrinth, after a while the agent repeatedly follows the same finite path, $\Pi_\circ$, again and again.

Proof:

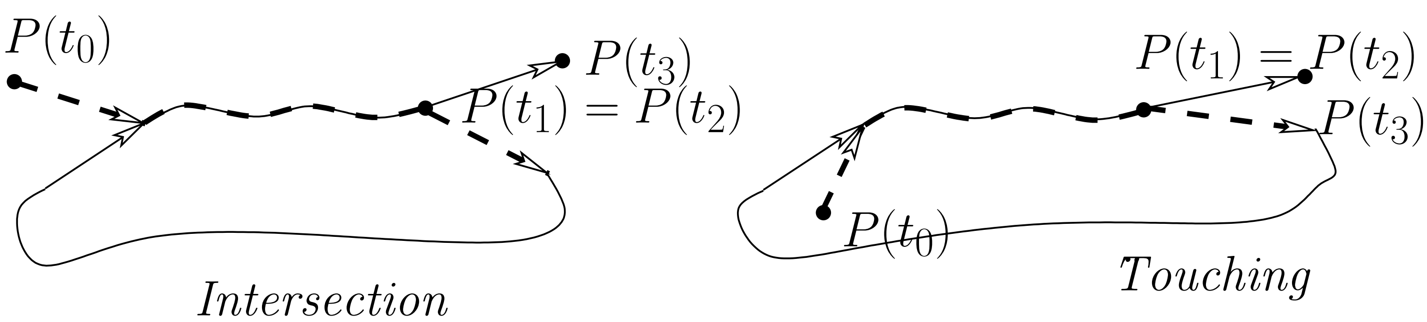
- Path is a polygonal chain
- Vertices I: Vertices of the polygons
- Vertices II: Hit-Points on the edges
- Correspond to vertices of type I
- Finite set S of possible vertices of the path
- The same counter value at the same vertex $\Rightarrow$ the same path again and again
- Assume: Never the same value

- Case 1: After a while, keeping on the boundary $\Rightarrow$ always the same path along one polygon
- Case 2: Leaving more than $|S|$ times (infinitely often)
- $\Rightarrow$ at least twice with the same value 0 at the same vertex, contradiction!

Correctness: $\Pi_\circ$ no self-intersection

Lemma Asumme the agent does not leave the labyrinth by Pledge and let $\Pi_\circ$ be the repeated path. $\Pi_\circ$ has no self-intersections.

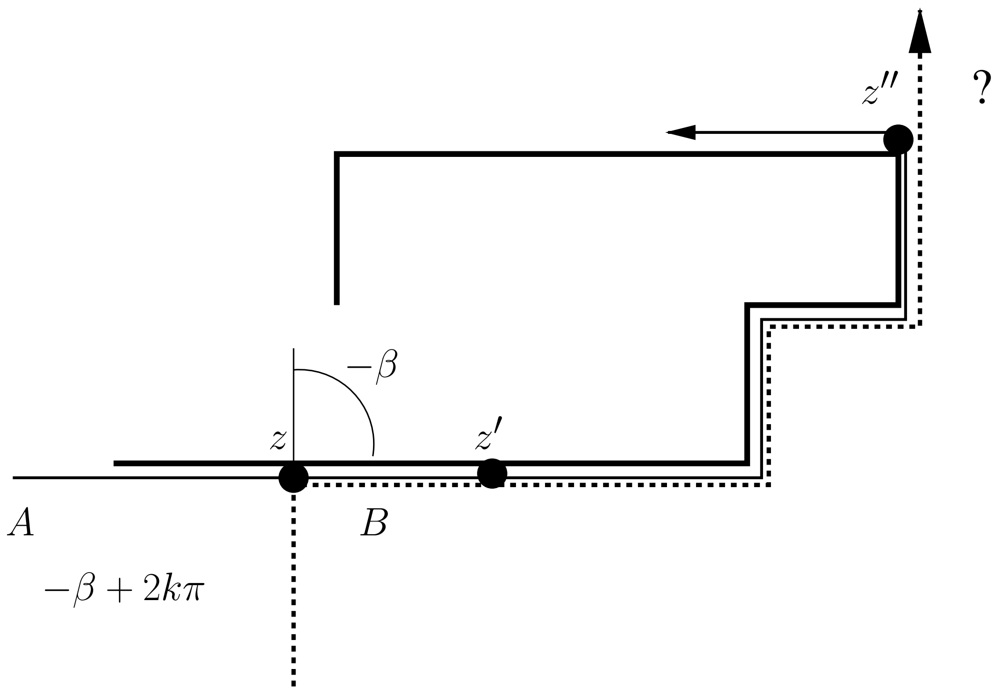
Difference: Intersection/Touching



Intersection only at the boundary! All *free paths* run in parallel!

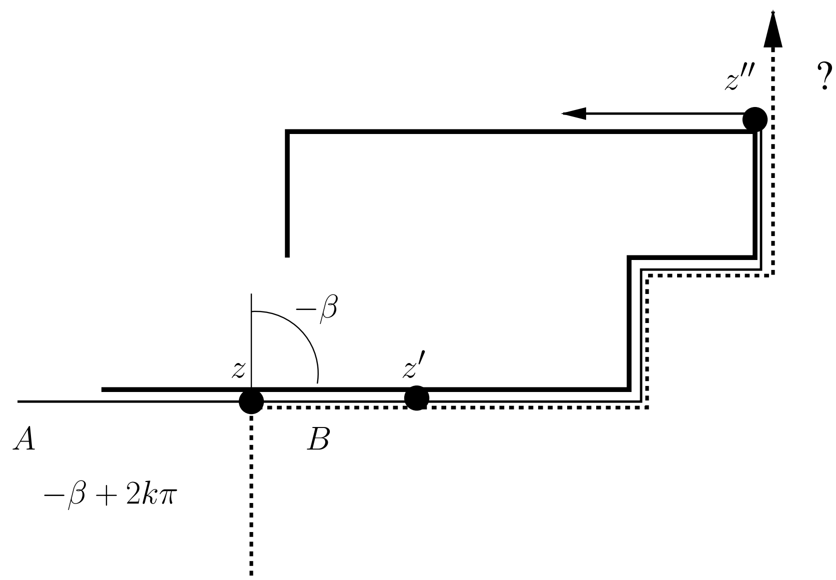
Correctness: $\Pi_\circ$ no self-intersection

- Proof: Ass. Intersection! Two parts one of which is free, say B
- Shortly behind z angular counter $C_A(z')$, $C_B(z')$
- $C_B(z') = -\beta$ and $C_A(z') = -\beta + 2k\pi$ for $k \in Z$



Correctness: $\Pi_\circ$ no self-intersection

- $C_B(z') = -\beta$ and $C_A(z') = -\beta + 2k\pi$ for $k \in Z$
- $k = 0$? A and B are the same! Contradiction!
- $k > 0$? Lemma, $C_A(z')$ negative
- Means $k < 0$ and $C_A(p) < C_B(p)$ for all p from z' to z''
- Path B leaves the obstacle first, no intersection!!!



Correctness proof

Theorem For any labyrinth and any starting position the pledge-algorithm will leave the labyrinth, if this is possible.

Proof:

- Ass.: Agent does not reach the boundary
- **Lemma** Path $\Pi_\circ$ again and again
- **Lemma** No intersections
- Orientations of $\Pi_\circ$: 1) cw-order 2) ccw-order
- 2) $+2\pi$ per full round, finally positive, contradiction
- Means 1) -2π per full round
- Remains negative after a while. Moves around obstacle!
- Orientation: cw-order, Left-Hand $\Rightarrow$ Enclosed!

Pledge algorithm with sensor errors

- Possible errors?
- Left-Hand-Rule, stable!
- Counting rotational angles!
- Hold the direction in the free space!
- For example: Compass!
- Full turns ok, but not precisely!
- Leave the obstacle slightly too early or too late!
- The main direction can be hold!
- Still correct?