Recap:

WSPD w.r.t. s

$(A_1, B_1), (A_2, B_2), \ldots, (A_m, B_m)$

Theorem 19 $m \in O(s^d d^{\frac{d}{2}} n)$

comp. time $O(d \cdot n \log n + s^d d^{\frac{d}{2}} \cdot n)$

T-Spanner $p, q \in S$ $|\pi_{G_p}^{q}| \leq t|pq|$

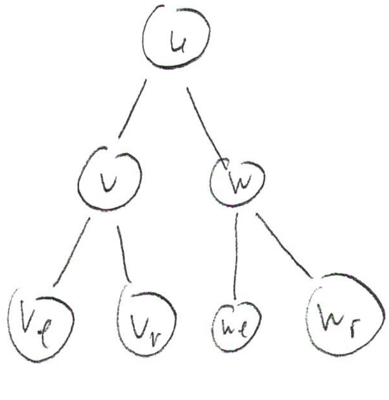
WSPD w.r.t. s $\xrightarrow[\text{Spanner constr.}]{}$ $t = \frac{s+4}{s-4}$

Construction WSPD

(Partial) Split Trees $\rightsquigarrow$ Split Tree

Find Pairs Procedure

FindPairs (v, w)



$L := \max\{L_{max}(R(S_v)), L_{max}(R(S_w))\}$

$r = \frac{\sqrt{d}}{2} L$

If (S_v, S_w) well-sep. w.r.t. s report (v, w)

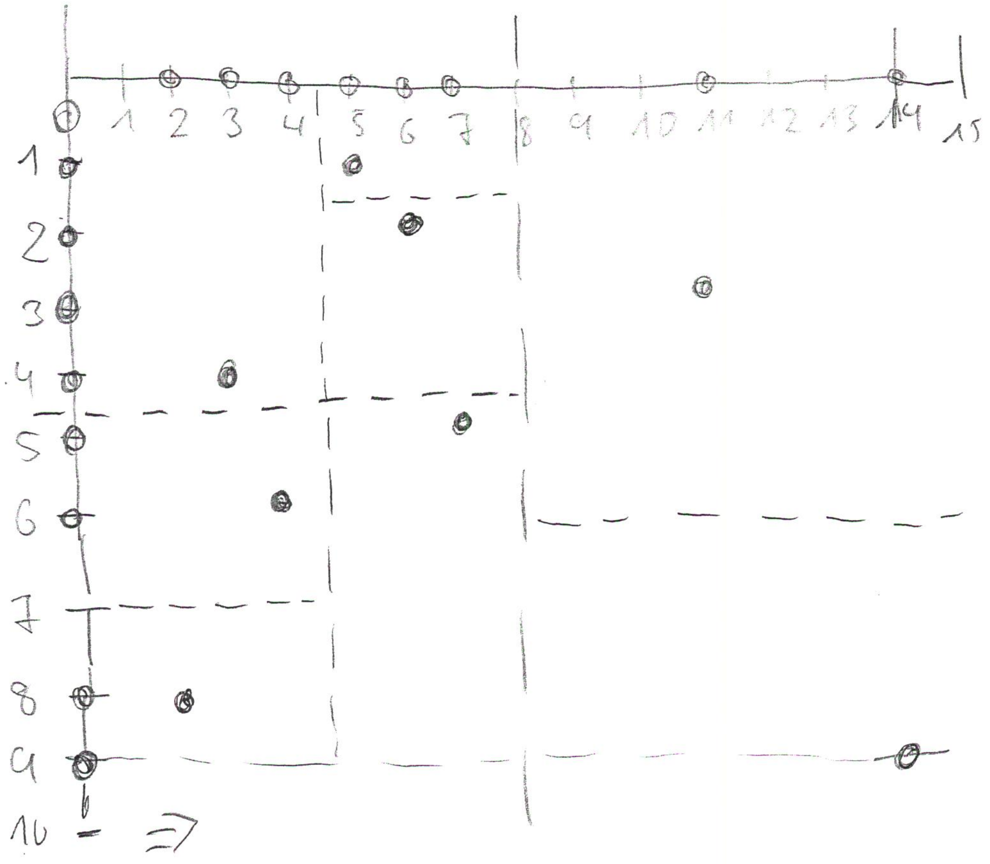
Else if $L_{max}(R(S_v)) > L_{max}(R(S_w))$

Then FindPairs (v_ℓ, w), FindPairs (v_r, w)

Else FindPairs (v, w_ℓ), FindPairs (v, w_r)

Partial

Spat Tree Constructn

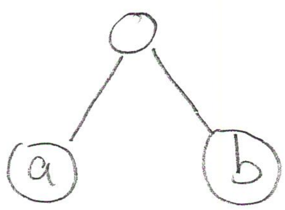


1. (2,8) (3,4) (4,6) (5,1) (6,2) (7,5) (11,3) (14,9) S^1

2. (5,1), (6,2) (11,3) (3,4) (7,5) (4,6) (2,8) (14,9) S^2

$L_{max} = 12$

$x = 8$



- Use copies of the lists
- Cross references between points!
- Doubly connected $\Leftrightarrow$ (7,5) $\Leftrightarrow$ (11,3)

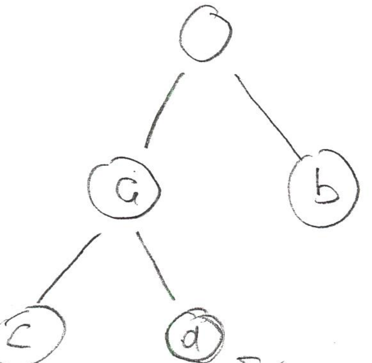
(7,5) → (4,6)

{(11,3), (14,9)}

Step 1 Find points (11,3), (14,9) Hyperplane $x=8$

- 2 Steps from right ⇝ Done leaf!
- Delete points {11,3} and {14,9} out of copies for this phase!
- Copies contains cross references

Step 2 for a



$\{(11,3),(14,9)\}$

$\{(5,1),(6,2)(3,4)\}$ $\{(7,5),(4,6),(2,8)\}$

$(2,8),(3,4),(4,6),(5,1),(6,2),(7,5)$

$(5,1),(6,2),(3,4),(7,5),(4,6),(2,8)$

Cross references to original list is still there!

$L_{max} = 7$ $\quad$ $Y = 4{,}5$

3 Steps

Use original Sets!

Leafs u and sets L_u

Initialize lists L_u^1 L_u^2 for $c, d, b = u$

For 1. (and 2.)

move through original lists and insert points in L_c^1, L_d^1, L_b^1 at the end of the lists!

Example: $L_c^1 = (3,4)\ (5,1)\ (6,2)$ and so on!

$L_d^1 = (2,8)$

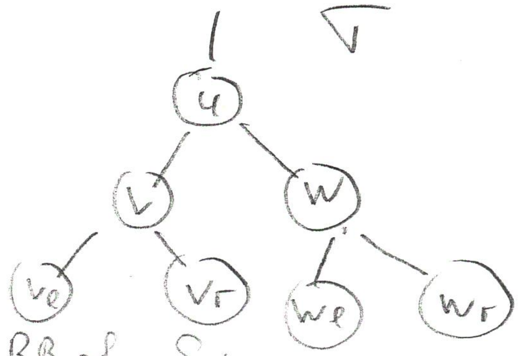
$L_b^1 = (4,6)$

Computing a WSPD of S w.r.t. s

Given Split Tree T of S, $s > 0$, $s \in \mathbb{R}$

For each internal node $u \in T$ with children v, w

invoke procedure



FindPairs(v,w)

$L_{max}(R(S_v))$ longest dimension of BB of S_v

(well-separated w.r.t. bounding boxes $R(S_v) \in O(d)$
w.r.t "radius" $\frac{L_{max}(R(S_v))}{2} \geq \frac{L_{max}(R(S_w))}{2}$)

$r = \frac{\sqrt{d}}{2} L$

FindPairs(v,w)

If S_v, S_w are well separated w.r.t s report (v,w)

(S_v, S_w)

Else if $L_{max}(R(S_v)) \geq L_{max}(R(S_w))$

Then FindPairs(v_ℓ, w), FindPairs(v_r, w)

Else FindPairs(v, w_ℓ), FindPairs(v, w_r)

Procedure terminates: Sets get smaller, for example

$|S_v| \cdot |S_{w_\ell}| < |S_v| \cdot |S_w|$ and $|S_v| \cdot |S_{w_r}| < |S_v| \cdot |S_w|$

$L_{max}(R(S_w)) \geq L_{max}(R(S_v))$

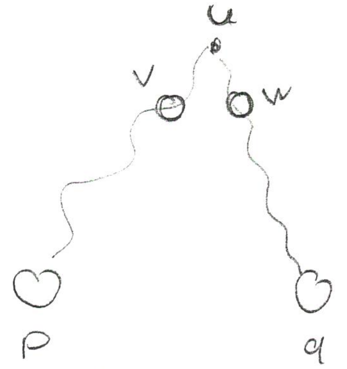
Lemma 20 The given procedure results in a WSPD w.r.t. S

$\{S_{v_1}, S_{w_1}\}, \{S_{v_2}, S_{w_2}\}, \ldots, \{S_{v_m}, S_{w_m}\}$

Proof: **Terminates** → Leafs has size 1. well-separated

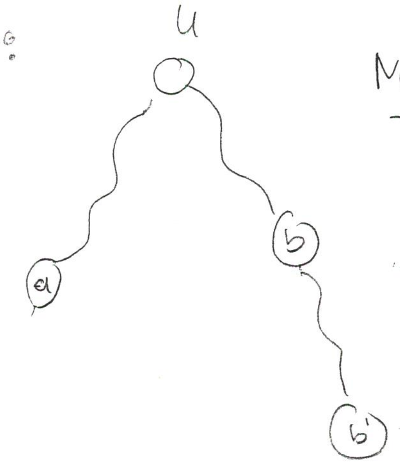
p, q distinct points of S

Exists $\{S_{v_i}, S_{w_i}\}$ with $p \in S_{v_i}$, $q \in S_{w_i}$?



Backwards from the leafs, lowest common ancestor u of p and q exists. guarantees existence! (lca)

Uniqueness:



Never reports!

(a, b) and (a, b') ↯

$(a,b)\ (a,b') \Rightarrow S_b \cap S_{b'} = \emptyset$

i, j vertices of $T \Rightarrow S_i \subset S_j$ or $S_j \subset S_i$ or $S_i \cap S_j = \emptyset$

□

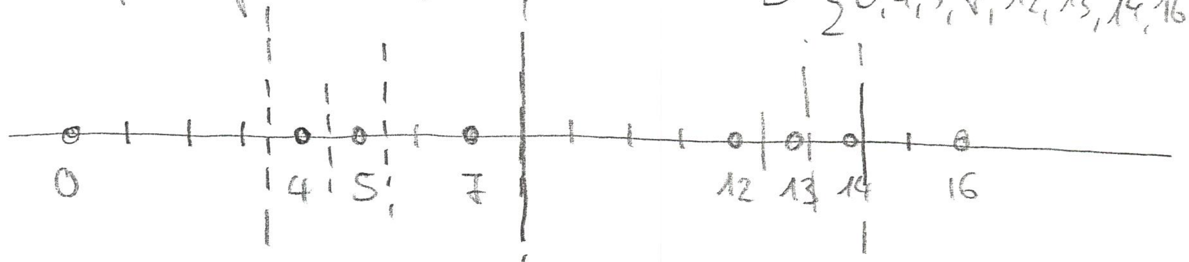
Not clear: number of pairs, m, reported

There are only $O(n)$ subsets S_a for the inner nodes $a \in T$,

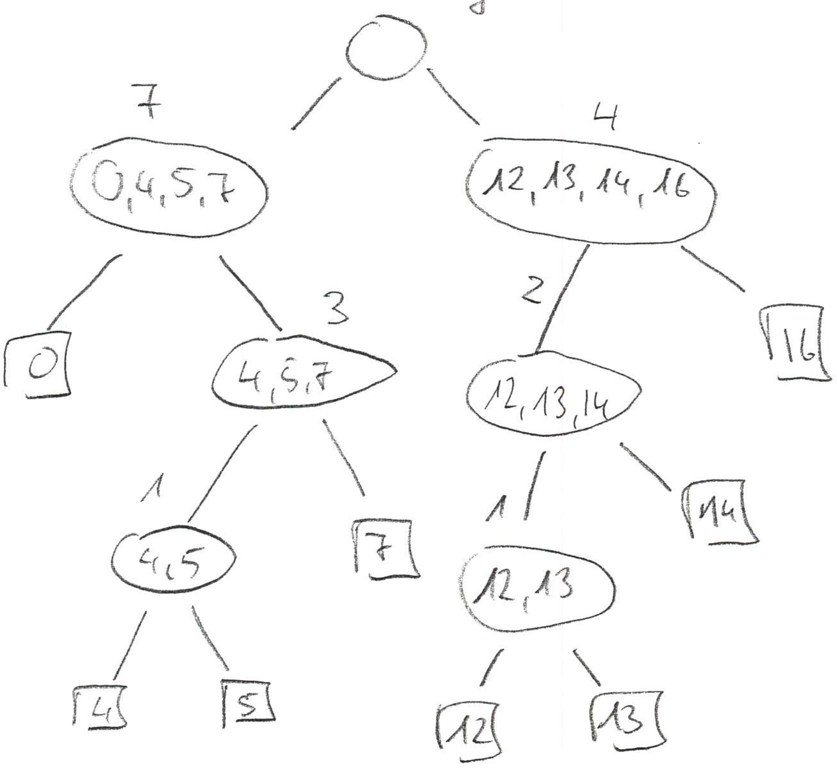
but the same set can occur many times

as A_i, A_j, B_k, B_l etc.

Example for WSPD, $s=3$ $\quad S=\{0,4,5,7,12,13,14,16\}$



Split Tree $T(S)$

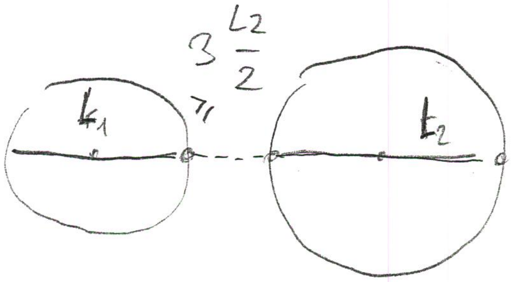


$D_{im} = 1$ $\quad L_{max}$ = length of the set

$\frac{\sqrt{d}}{2} L$ $\quad \frac{L}{2}$ Radius of smallest enclosing "disk"

A, B well separated w.r.t $s=3$

$\Leftrightarrow$



$L_1 < L_2$

Distance $\geq \frac{3}{2}$ max length

WSPD - Computation via Find Pairs (Forest, Collection of Trees)

$FP(\overset{7}{\{0,4,5,7\}}, \overset{4}{\{12,13,14,16\}})$

$7+3\cdot\frac{7}{2} > 12$

$FP(\overset{0}{\{0\}}, \overset{4}{\{12,13,14,16\}})$ ✓

$0+3\cdot\frac{4}{2} \leq 12$

$FP(\overset{3}{\{4,5,7\}}, \overset{4}{\{12,13,14,16\}})$

$7+3\frac{4}{2} > 5$

$FP(\overset{3}{\{4,5,7\}}, \overset{2}{\{12,13,14\}})$

$7+3\cdot\frac{3}{2} \leq 12$

$FP(\overset{3}{\{4,5,7\}}, \overset{0}{\{16\}})$

$7+3\cdot\frac{3}{2} \leq 12$ ✓

$FP(\overset{0}{\{0\}}, \overset{3}{\{4,5,7\}})$

$0+3\cdot\frac{3}{2} > 4$

$FP(\overset{0}{\{0\}}, \overset{1}{\{4,5\}})$ ✓ $-$ $FP(\{0\}, \{7\})$ ..

$0+3\cdot\frac{1}{2} \leq 4$

$FP(\overset{2}{\{12,13,14\}}, \overset{0}{\{16\}})$

$14+3\cdot 1 \not> 16$

$FP(\overset{1}{\{4,5\}}, \overset{0}{\{7\}})$

$5+3\frac{1}{2} \leq 7$ ✓

$FP(\overset{1}{\{12,13\}}, \overset{0}{\{14\}})$

$13+3\cdot\frac{1}{2} > 14$

$FP(\overset{1}{\{12,13\}}, \overset{0}{\{16\}})$ ✓

$13+3\cdot\frac{1}{2} \leq 16$

$FP(\{14\}, \{16\})$ ✓

$FP(\{12\}, \{14\})$ ✓ $FP(\{13\}, \{14\})$ ✓

$FP(\{4\}, \{5\})$ ✓

$FP(\{12\}, \{13\})$ ✓