Discrete and Computational Geometry, SS 18
Exercise Sheet "1": Dilation of Graphs and Chains
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 17th of April**.
- *You may work in groups of at most two participants.*
- *You can hand over your work to our tutor Raoul Nicolodi in the beginning of the lecture.*

Exercise 1: Number of dilation pairs (4 Points)

The geometric dilation of a planar graph G is defined as

$$\delta_{geom}(G) := \sup_{p \neq q \in G} \frac{|\pi_p^q|}{|pq|}$$

where $|pq|$ is the euclidean distance from p to q and $|\pi_p^q|$ is the length of a shortest path in G from p to q.

Let Γ be the set of non-intersecting polygonal chains C in the plane where $\delta(C) > 1$. Let n be the number of vertices of C and $P(C)$ be the number of pairs $p, q \in C$, where the geometric dilation of C is attained and p is a vertex of C.

- Prove the upper bound $P(C) \in O(n^2)$ if $C \in \Gamma$.
- Verify that $P(C) \in \Omega(n^2)$ holds by giving a construction scheme for suitable chains C (for an arbitrary number n of vertices).
- Prove $P(C) = \infty$ holds if $C \notin \Gamma$ is an arbitrary non-intersecting polygonal chain.

Exercise 2: Visibility and maximum dilation (4 Points)

The graph-theoretic dilation of a planar graph G with vertex set V is

$$\delta_{graph}(G) := \sup_{p \neq q \in V} \frac{|\pi_p^q|}{|pq|}$$

where $|pq|$ is the euclidean distance from p to q and $|\pi_p^q|$ is the length of a shortest path in G from p to q.

- Construct a planar graph G where the maximum graph-theoretic dilation of G is attained by a pair of non-visible vertices.
- Recall the definition of geometric dilation of a planar graph. Prove that for a planar, simply connected graph G there is always a pair of points $p, q \in G$ with maximal dilation so that p and q are co-visible.

Exercise 3: Dilation and AVDs (4 Points)

The decision problem for the geometric dilation of a polygonal chain $C = (p_1, p_2, \ldots, p_n)$ was translated into the problem of tracing the chain C through an additively weighted Voronoi diagram.

We proved the following statement: *If for a point $(q_x, q_y) \in C$ appearing after $C_i = (p_1, p_2, \ldots, p_i)$ on C, the point (q_x, q_y, a_q) with $a_q := \frac{|C_{p_1}^q|}{K}$ lies below any cone K_{p_i} starting at height $a_{p_i} := \frac{|C_{p_1}^{p_i}|}{K}$ at p_i, the dilation $\delta_C(p_i, q)$ between q and p_i is smaller than K.*

- Why do we trace the chain $p_i, p_{i+1}, \ldots, p_n$ through the additively weighted Voronoi diagram of $p_1, p_2, \ldots, p_i$ with weigths a_{p_i}?
- Why can we compute the Voronoi diagram for all points $p_1, p_2, \ldots, p_n$ with weights a_{p_i} and trace the complete chain (for one direction) only once? Or the other way round: Why is it not necessary to incrementally compute the Voronoi diagrams for $p_1, p_2, \ldots, p_i$ and successively trace the chains $p_i, p_{i+1}, \ldots, p_n$?