Introduction to Computational Topology
Summer semester 2018

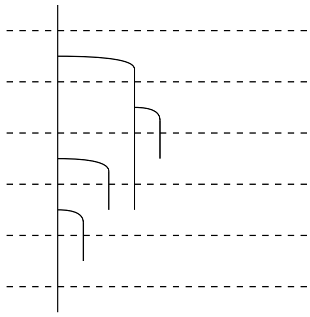
Discussion: 27.06. - 29.06.

Prof. Dr. Rolf Klein
Barbara Schwarzwald
Institute of Computer Science I

Exercise Sheet 8

Exercise 8.1: Elder Rule effects (4 Punkte)

Consider the following path decomposition of a merge tree generated from a filtration based on the elder rule - where the dashed lines symbolize the different levels of the filtration and the bending path at an intersection is the path that ended upon two components merging.



How does the path decomposition for the same filtration look like if instead of using the elder rule, a contrasting 'younger rule' is used, where the younger path is kept?? How many paths span from a level a all the way through to a level b?

Exercise 8.2: Fundamental Lemma (4 Punkte)

Prove the following fundamental Lemma of persistent homology mentioned in the lecture:

Given the following definitions for a filtration $\emptyset = K_0 \subseteq K_1 \subseteq \ldots \subseteq K_n = K$

$$H_p^{i,j} := Imh_p^{i,j} = Z_p(K_i)/(B_p(K_j) \cap Z_p(K_i)) \subseteq H_p(K_j)$$

$$\beta_p^{i,j} := dim H_p i, j \qquad p^{th} \text{persistent Betti number}$$

$\lambda_p^{i,j} :=$ number of p-classes born at K_i and dying at K_j,

so (since $\beta_p^{i,j-1} - \beta_p^{i,j}$ is the number of p-classes born at $\leq K_i$ and dying at K_j), while $\beta_p^{i-1,j-1} - \beta_p^{i-1,j}$ is the number of p-classes born at $\leq K_{i_1}$ and dying at K_j)

$$\lambda_p^{i,j} = \beta_p^{i,j-1} - \beta_p^{i,j} - (\beta_p^{i-1,j-1} - \beta_p^{i-1,j})$$

Then it holds for $0 \leq k \leq l \leq n$:

$$\beta_p^{k,l} = \sum_{i \leq k} \sum_{j < l} \lambda_p^{i,j}$$

Exercise 8.3: Complexity of pseudodisc polygon union (4 Punkte)

Consider the following situation.

n convex polygons (each of constant complexity) form a family of pseudodiscs, i.e. each pair of polygons have at most 2 intersections.

What is the complexity of the border of the union of this polygons?