Discrete and Computational Geometry, WS1516
Exercise Sheet "5": Abstract Voronoi Diagrams
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Friday 4th of December, 12:00 pm**.
- *There is a letterbox in front of Room E.01 in the LBH builiding.*
- *You may work in groups of at most two participants.*

Exercise 11: Line segments and Abstract Voronoi diagram (4 Points)

Consider a set S of n disjoint line segments, and let $\mathcal{J}$ be the $\binom{n}{2}$ bisecting curves among S. Please prove the bisecting system $(S, \mathcal{J})$ is admissible, i.e., the corresponding Voronoi diagram is an abstract Voronoi diagram. (Prove the following three Axioms).

(A1) Each bisecting curve in $\mathcal{J}$ is homeomorphic to a line (not closed)

(A2) For each non-empty subset S' of S and for each $p \in S'$, $\mathrm{VR}(p, S')$ is path-connected.

(A3) For each non-empty subset S', $R^2 = \bigcup_{p \in S'} \overline{\mathrm{VR}(p, S')}$

Exercise 12: Karlsruhe metric (4 Points)

The Karlsruhe metric, also known as the Moscow metric, is a distance measure in a radial city where there is a city center, and roads either circumvent the center or are extended from the center. The distance $d_K(p_1, p_2)$ between two points is $\min(r_1, r_2) \times \delta(p_1, p_2) + |r_1 - r_2|$ if $0 \le \delta(p_1, p_2) \le 2$ and $r_1 + r_2$, otherwise, where (r_i, ψ_i) are the polar coordinates of p_i with respect to the center, and $\delta(p_1, p_2) = \min(|\psi_1 - \psi_2|, 2\pi - |\psi_1 - \psi_2|)$ is the angular distance between the two points. Please prove the bisecting curve system in the Karlsruhe metric to be admissible. (Assume that there is no point equidistant from four sites).

Bonus 2: Transitivity (8 Points)

Let $\mathcal{J}$ be an admissible system satisfying the following axioms

(A1) Each bisecting curve in $\mathcal{J}$ is homeomorphic to a line (not closed)

(A2) For each non-empty subset S' of S and for each $p \in S'$, $\mathrm{VR}(p, S')$ is path-connected.

(A3) For each non-empty subset S', $R^2 = \bigcup_{p \in S'} \overline{\mathrm{VR}(p, S')}$

Assume that any two p-bisectors $J(p, q)$ and $J(p, r)$ intersect at most two points and the intersections are transversal. Please prove

$$\overline{D(p,q)} \cap \overline{D(q,r)} \subseteq \overline{D(p,r)}.$$