Discrete and Computational Geometry, WS1516 Exercise Sheet "8": Lattices

University of Bonn, Department of Computer Science I

- Written solutions have to be prepared until Friday 22th of January, 12:00 pm.
- There is a letterbox in front of Room E.01 in the LBH building.
- You may work in groups of at most two participants.

Exercise 15: Integer Lattices (4 Points)

Prove: If $C \subseteq \mathbb{R}^d$ is convex, symmetric around the origin, bounded, and such that $\operatorname{vol}(C) > k \cdot 2^d$, then C contains at least 2k lattice points.

Exercise 16: Determinant and Volume of Parallelepiped (4 points)

Let v_1, \ldots, v_d be linearly independent vectors in \mathbb{R}^d . Form a matrix A with v_1, \ldots, v_d as rows. Prove that $|\det A|$ is equal to the volume of the parallelepiped $\{\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_d v_d \mid \alpha_1, \cdots, \alpha_d \in [0, 1]\}.$