

Theoretical Aspects of Intruder Search

Course Wintersemester 2015/16
Escape Paths for the Intruder

Elmar Langetepe

University of Bonn

January 26th, 2016

Reverse Situation

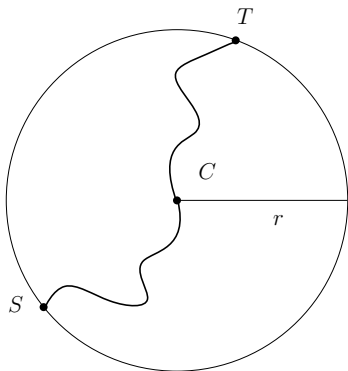
- Take over the position of the intruder
- Try to escape from an partially unknown environment
- The adversary manipulates the environment
- Leave the area as soon as possible
- *Lost in a forest* Bellman 1956
- Escape paths for region R
- Single deterministic path
- Leave area from any starting point
- Adversary translates and rotates R
- Minimize the length of successful path
- Geometric argumentations
- Only known for few shapes

Simple examples

Obviously: The diameter of any region R is always an escape path!

Theorem 69: The shortest escape path for a circle of radius r is a line segment of length $2r$.

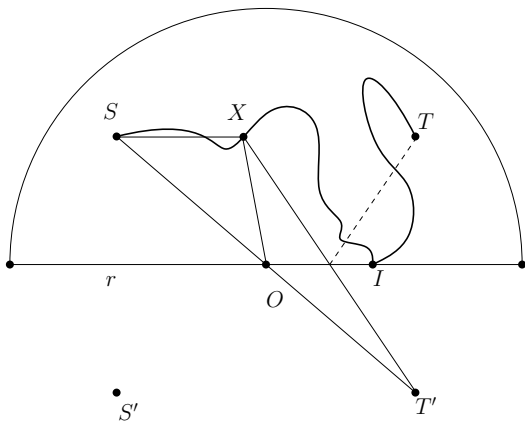
Proof: Assume there is a better escape path! Contradiction!



Also for semicircles

Theorem 70: The shortest escape path for a semicircle of radius r is a line segment of length $2r$.

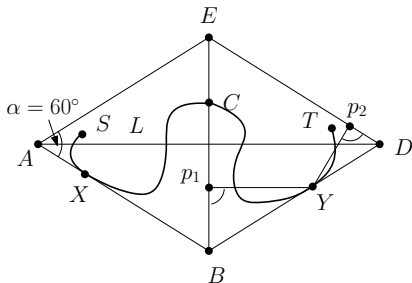
Proof: Assume there is a better escape path! Contradiction!



More generally for a rhombus with angle 60°

Theorem 71: The shortest escape path for rhombus of diameter L with angle $\alpha = 60^\circ$ is a line segment of length L .

Proof: Assume there is a better escape path! Contradiction!

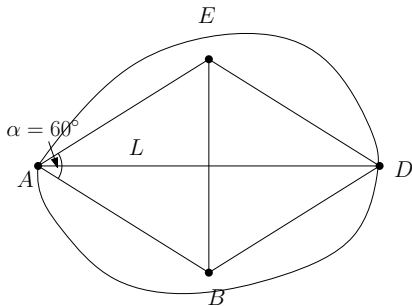


Fatness definition!

Definition: Fatness w.r.t. diameter! Rhombus-Fat!

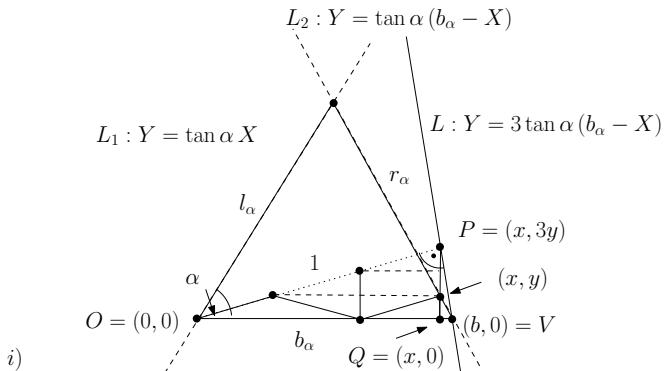
Corollary: The shortest escape path for rhombus-fat convex set of diameter L is a line segment of length L .

Proof: Assume there is a better escape path! Contradiction!



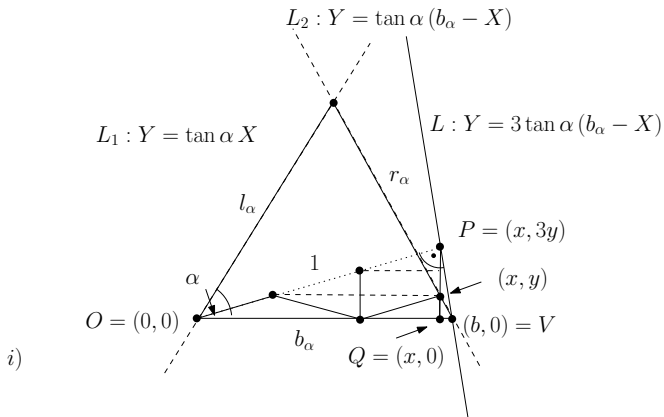
Convex = diameter?

- Equilateral triangle: Besicovitch
- Zig-Zag escape path with length ≈ 0.9812
- More generally from Coulton and Movshovich (2006)
- Isosceles triangle for α and b_α
- b_α is diameter!



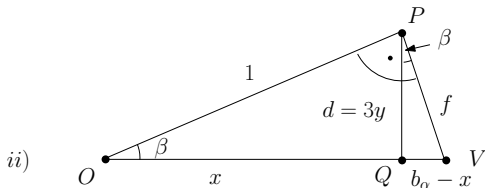
Convex = diameter?

- Construct symmetric Zig-Zag path of small length
- Assume length 1.



Convex = diameter?

- Extract triangle
- $\frac{1}{x} = \frac{b_\alpha}{1} \quad x = \frac{1}{b_\alpha}$

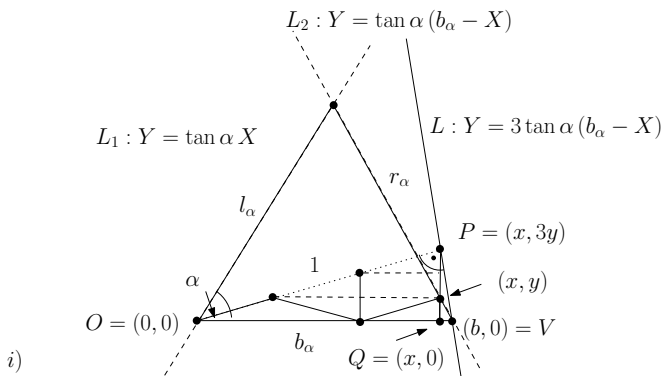


Convex = diameter?

Finally we determine b_α :

$y = \tan \alpha \left(b_\alpha - \frac{1}{b_\alpha} \right)$ and $x = \frac{1}{b_\alpha}$ and $x^2 + (3y)^2 = 1$ which gives

$$b_\alpha = \sqrt{1 + \frac{1}{9 \tan^2 \alpha}}.$$

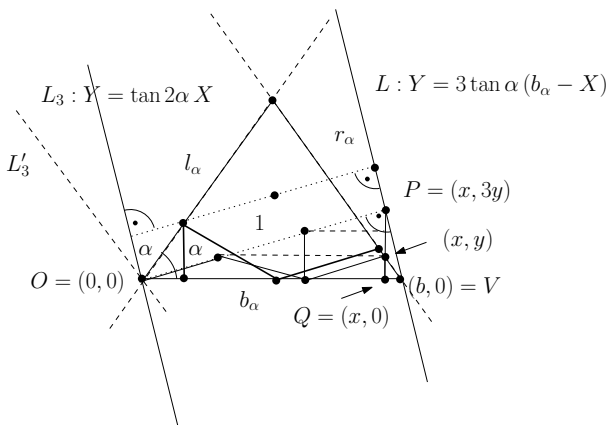


Further constraint for α

There should be no better Zig-Zag path for T_α !

Line $L_3 : Y = \tan(2\alpha) X$ runs in parallel with L_2 . This means

$$-3 \tan \alpha = \tan 2\alpha \text{ or } \tan \alpha = \sqrt{\frac{5}{3}}.$$

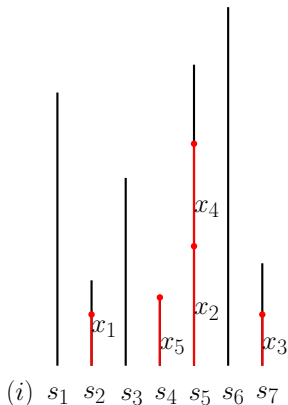


Theorem 72: For any $\alpha \in [\arctan(\sqrt{\frac{5}{3}}), 60^\circ]$ there is a symmetric Zig-Zag path of length 1 that is an escape path of T_α smaller than the diameter b_α .

- $b_\alpha = \sqrt{1 + \frac{1}{9 \tan^2 \alpha}}$
- $\alpha = 60^\circ: b_\alpha = \sqrt{\frac{28}{27}}$
- $b_\alpha := 1 \implies \sqrt{\frac{27}{28}} < 1$ is Zig-Zag path length
- Optimality? Yes!

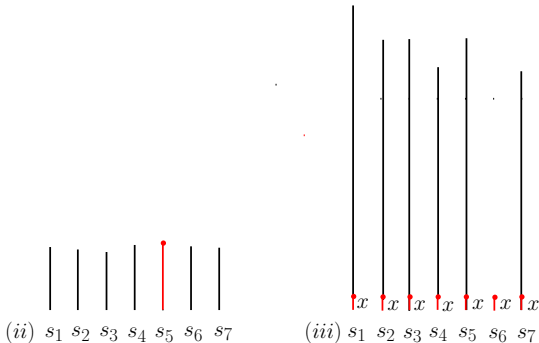
Different performance measures

- Set L_m of m line segments s_i of unknown length $|s_i|$
- Dark corridors, escape, digging for oil
- Test corridors successively
- s_{j_1} up to a certain distance x_1 , then s_{j_2} for another distance x_2 and so on



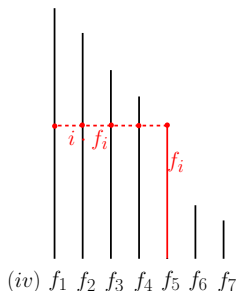
Length are known! Extreme Cases!

- Assume distribution is known!
- $f_1 \geq f_2 \geq \dots \geq f_m$ order of the length given
- Extreme cases! Good strategies!



Known length in general

- $f_1 \geq f_2 \geq \dots \geq f_m$ order of the length given
- Check i arbitrary segments with length f_i :
 $\min_i i \cdot f_i$ is a reasonable strategy
- $C(F_m, A)$ travel cost for algorithm A
- $\max\text{Trav}(F_m) := \min_A C(F_m, A)$

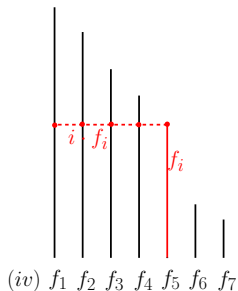


Optimal strategy for this case

Theorem 73: For a set of sorted distances F_m (i.e. $f_1 \geq f_2 \geq \dots \geq f_m$) we have

$$\max \text{Trav}(F_m) := \min_i i \cdot f_i.$$

Proof:



Optimal strategy for this case

Theorem 73: For a set of sorted distances F_m (i.e. $f_1 \geq f_2 \geq \dots \geq f_m$) we have

$$\max\text{Trav}(F_m) := \min_i i \cdot f_i.$$

Proof:

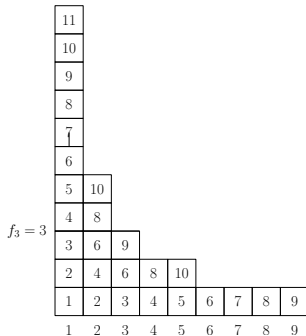
- Arbitrary strategy A
- Less than $\min_i i \cdot f_i$ means less than $j \cdot f_j$ for any j
- Visiting depth $d_1 \geq d_2 \geq \dots \geq d_m$
- Not reached f_1 by d_1 ,
not reached f_2 , since $d_1 + d_2 < 2f_2$ and $d_2 \leq d_1$ and so on
- Not successful!
- $\min_i i \cdot f_i$ always sufficient!

Online Strategy

- F_m with $f_1 \geq f_2 \geq \dots \geq f_m$ not known
- Compete against $\max\text{Trav}(F_m) := \min_i i \cdot f_i$
- Dovetailing strategy: Rounds $c = 1, 2, 3, 4, \dots$
- For any round c from left to right:
Path length of segment i is *extended* up to distance $\lfloor \frac{c}{i} \rfloor$

Online Strategy

- Dovetailing strategy: Rounds $c = 1, 2, 3, 4, \dots$
- For any round c from left to right:
Path length of segment i is *extended* up to distance $\lfloor \frac{c}{i} \rfloor$



Theorem 74: Hyperbolic traversal algorithm solves the multi-segment escape problem for any list F_m with maximum traversal cost bounded by

$$D \cdot (\max \text{Trav}(F_m) \ln(\min(m, \max \text{Trav}(F_m))))$$

for some constant D .

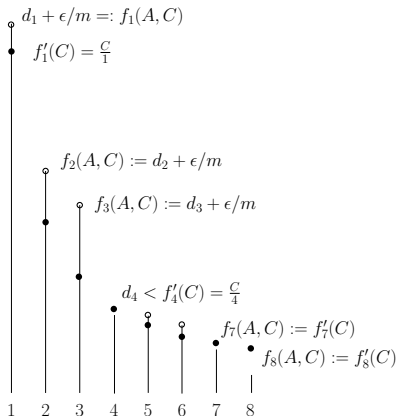
Proof:(W.l.o.g. F_m integers)

- Let $\min_i i \cdot f_i = j \cdot f_j$ for some j
- c with $c = j \cdot f_j$ exists (Round c)
- Overall cost:

$$\sum_{t=1}^m \left\lfloor \frac{c}{t} \right\rfloor \leq \sum_{t=1}^{\min(m,c)} \frac{c}{t} \leq c + \int_1^{\min(m,c)} \frac{c}{t} dt = c(1 + \ln \min(m, c)).$$

Matches Lower bound!

Theorem 75: For any deterministic online strategy A that solves the multi-segment escape problem we can construct input sequences $F_m(A, C)$ so that A has cost at least $d \cdot C \ln \min(C, m)$ and $\max \text{Trav}(F_m(C, A)) \leq C$ holds for some constant d and arbitrarily large values C .



Matches Lower bound! Proof!

- C is given! $f'_i(C) = \frac{C}{i}$ (not yet fixed)
- Wait until cost $\sum_{i=1}^m d_i \geq d \cdot C \ln \min(C, m)$ for some d
- Fix the scenario as shown below!

