Discrete and Computational Geometry, SS 14
Exercise Sheet “1”: Dilation of Graphs and Chains
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 22nd of April, 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

**Exercise 1: Number of dilation pairs (4 Points)**

The geometric dilation of a planar graph $G$ is defined as

$$\delta(G) := \sup_{p \neq q \in G} \frac{|\pi_p^q|}{|pq|}$$

where $|pq|$ is the euclidean distance from $p$ to $q$ and $|\pi_p^q|$ is the length of a shortest path in $G$ from $p$ to $q$.

Let $\Gamma$ be the set of non-intersecting polygonal chains $C$ in the plane where $\delta(C) > 1$. Let $n$ be the number of vertices of $C$ and $P(C)$ be the number of pairs $p, q \in C$, where the geometric dilation of $C$ is attained and $p$ is a vertex of $C$.

- Prove the upper bound $P(C) \in O(n^2)$ if $C \in \Gamma$.
- Prove $P(C) = \infty$ holds if $C \notin \Gamma$ is an arbitrary non-intersecting polygonal chain.
- Verify that $P(C) \in \Omega(n^2)$ holds by giving a construction scheme for suitable chains $C$ (for an arbitrary number $n$ of vertices).

**Exercise 2: Visibility and maximum dilation (4 Points)**

The graph-theoretic dilation of a planar graph $G$ with vertex set $V$ is

$$\delta_{graph}(G) := \sup_{p \neq q \in V} \frac{|\pi_p^q|}{|pq|}$$

where $|pq|$ is the euclidean distance from $p$ to $q$ and $|\pi_p^q|$ is the length of a shortest path in $G$ from $p$ to $q$.

- Construct a planar graph $G$ where the maximum graph-theoretic dilation of $G$ is attained by a pair of non-visible vertices.
- Recall the definition of geometric dilation of a planar graph. Prove that for a planar, simply connected graph $G$ there is always a pair of points $p, q \in G$ with maximal dilation so that $p$ and $q$ are co-visible.

**Exercise 3: Dilation and AVDs (4 Points)**

The decision problem for the geometric dilation of a polygonal chain $C = (p_1, p_2, \ldots, p_n)$ was translated into the problem of tracing the chain $C$ through an additively weighted Voronoi diagram.

We proved the following statement: *If for a point $(q_x, q_y) \in C$ appearing after $C = (p_1, p_2, \ldots, p_i)$ on $C$, the point $(q_x, q_y, a_q)$ with $a_q := \frac{|C_{p_1}^q|}{K}$ lies below any cone $K_{p_i}$ starting at height $a_{p_i} := \frac{|C_{p_1}^{p_i}|}{K}$ at $p_i$, the dilation $\delta_C(p_i, q)$ between $q$ and $p_i$ is smaller than $K$.*

- Why do we trace the chain $p_i, p_{i+1}, \ldots, p_n$ through the additively weighted Voronoi diagram of $p_1, p_2, \ldots, p_i$ with weigths $a_{p_i}$?
- Why can we compute the Voronoi diagram for all points $p_1, p_2, \ldots, p_n$ with weights $a_{p_i}$ and trace the complete chain (for one direction) only once? Or the other way round: Why is it not necessary to incrementally compute the Voronoi diagrams for $p_1, p_2, \ldots, p_i$ and successively trace the chains $p_i, p_{i+1}, \ldots, p_n$?