Discrete and Computational Geometry, SS 14
Exercise Sheet "3": Randomized Algorithms for Geometric Structures
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 6th of May, 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 1: Average Complexity of Sorting (4 Points)

Given a set N of n real numbers, please analyze the average complexity for the following sorting algorithms over all the $n!$ permutation sequences of N.

- Insertion Sort
- Merge Sort
- Quick Sort (always select the first element)

Exercise 2: Trapezoidal Decomposition (4 Points)

Given a set N of n line segments with a total number k of intersections in the plane, let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please prove the following

- The vertical trapezoidal decomposition $H(N)$ of N has $O(n+k)$ trapezoids (faces) even if more than two line segments can intersects at the same point.

- The expected number of trapezoids in $H(N^i)$ is $O(i + ki^2/n^2)$. (Hint: the expected number of intersections)

Exercise 3: Triangulations (4 Points)

Given a set N of n points in the plane, a triangulation $T(N)$ of N is a maximal planar straight-light graph, i.e., every edge is a straight-line segment, and no edge can be added to main the planarity. Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $T(N)$ by computing $T(N^3), T(N^4), \ldots, T(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 3$, obtain $T(N^{i+1})$ from $T(N^i)$ by adding S_{i+1}.

1. Describe the insertion of S_{i+1}
2. Define a conflict relation between a triangle in $H(N^i)$ and a point in $N \setminus N^i$
3. Prove the expected cost of inserting S^{i+1} to be $O(\frac{n}{i+1})$ and the expected cost of construction $T(N)$ to be $O(n \log n)$