Discrete and Computational Geometry, WS1516
Exercise Sheet "2": Master Theorem and Voronoi Diagrams
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Wednesday 11th of November, 12:00 pm**.
- *There is a letterbox in front of Room E.01 in the LBH builiding.*
- *You may work in groups of at most two participants.*

Exercise 1: Master Theorem I (4 Points)

Consider a function $T(\cdot)$ satisfying the following recurrence:

$$T(n) = (\ln r + 1)T(\lceil \alpha n \rceil) + O(D(n)),$$

where r, $\alpha < 1$, and $\epsilon > 0$ are constants and $D(n)$ is a function such that $D(n)/n^{\epsilon}$ is monotone increasing in n. Please prove that if $(\ln r + 1)\alpha^{\epsilon} < 1$, $T(n) \leq C \cdot D(n)$, where C is a constant depending on r, α, and ϵ.

Exercise 2: Master Theorem II (4 Points)

Consider a function $T(\cdot)$ satisfying the following recurrence:

$$T(n) = 2T(\lceil \frac{n}{2} \rceil) + O(D(n)),$$

where $D(n)/n$ is monotone increasing in n and ϵ is a positive constant. Please prove the following.

- $T(n) = O(D(n) \log n)$.
- If $D(n)/n^{1+\epsilon}$ is monotone increasing in n where $\epsilon > 0$, $T(n) = O(D(n))$.

Exercise 3: Voronoi Diagrams (4 Points)

Given a set S of n points in the Euclidean plane, the Voronoi diagram $V(S)$ partitions the plane into Voronoi regions $\mathrm{VR}(p, S)$, $p \in S$, such that all points in $\mathrm{VR}(p, S)$ share the same nearest site p among S. We make a general position assumption that no more than three points of S are located on the same circle. Let e, v, and u be the numbers of edges, vertices, unbounded faces of $V(S)$.

1. Please prove $e = 3(n-1) - u$ and $v = 2(n-1) - u$. (Hint: use Euler's forumla)
2. Please explain that the number of vertices will not increase without the general position assumption. In other words, $v \leq 2(n-1) - u$.