

Online Motion Planning MA-INF 1314

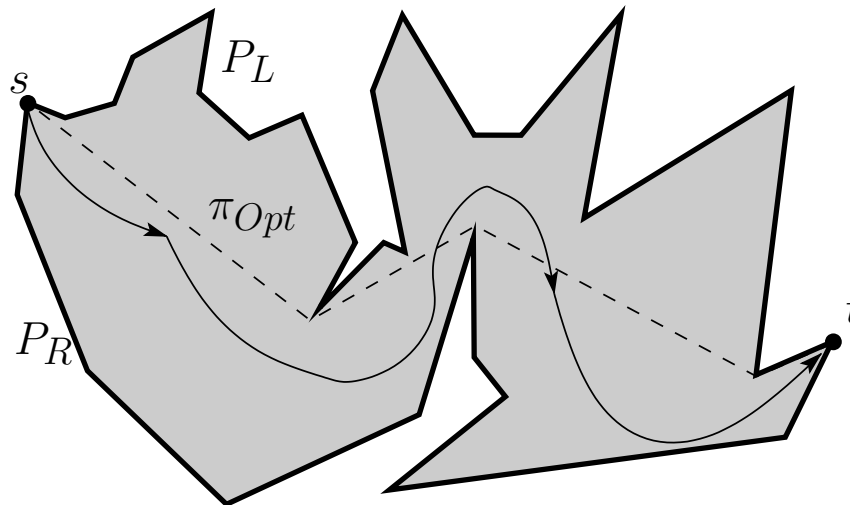
General rays! Searching in streets!

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Rep.: Street

Def. Polygonal boundary chains P_L and P_R of P between s and t weakly visible.

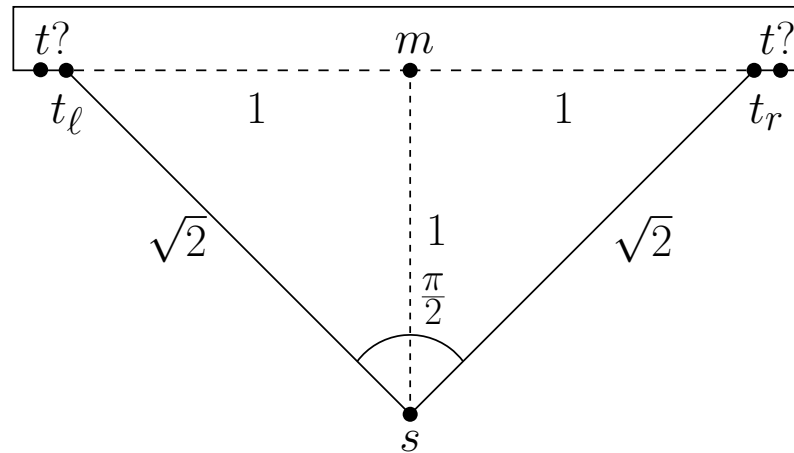
Task: Start at s , find t !



Rep.: Lower Bound!

Theorem: No strategy attains a ratio better than $\sqrt{2}$ versus the length of the shortest path. ■

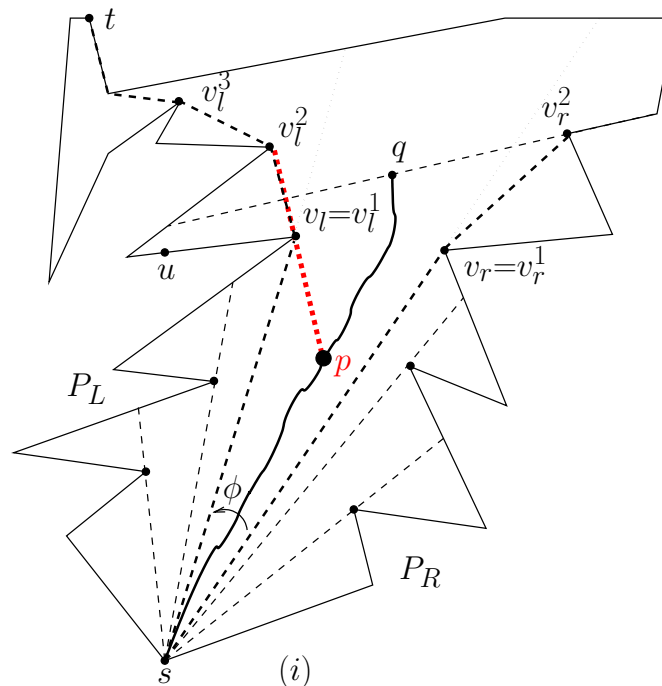
Beweis: ■



Detour of factor at least $\sqrt{2}$ ■

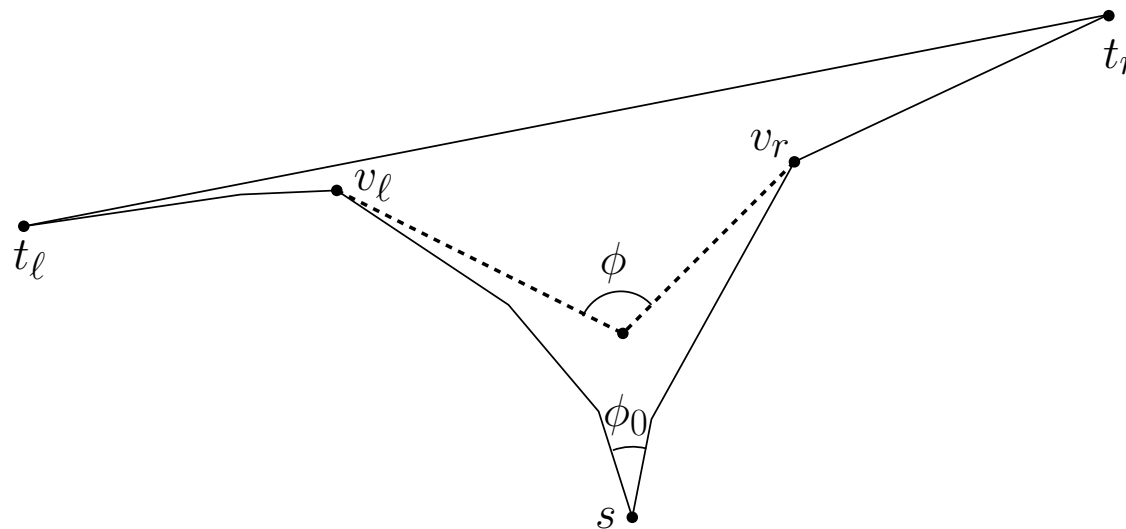
Rep.: Reasonable strategies!

- Rightmost left reflex vertex, leftmost right reflex vertex! ■
- Move into the wedge of c , v_l and v_r ■
- One side-candidate vanishes, move directly to the other ■
- Extreme vertices change over time ■



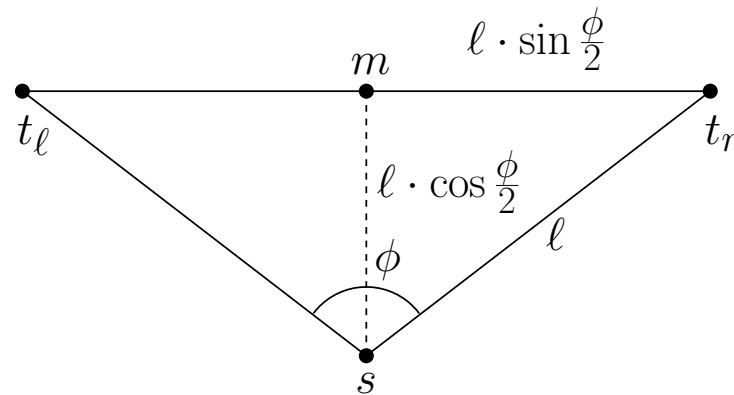
Rep.: Funnel polygons!

- It is sufficient to analyse special streets
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- **Def.** Polygon, convex vertex s , two opening convex polygonal chains P_L and P_R starting in s ending at t_ℓ and t_r , respectively. Segment $\overline{t_\ell t_r}$ closes the funnel (polygon). ■



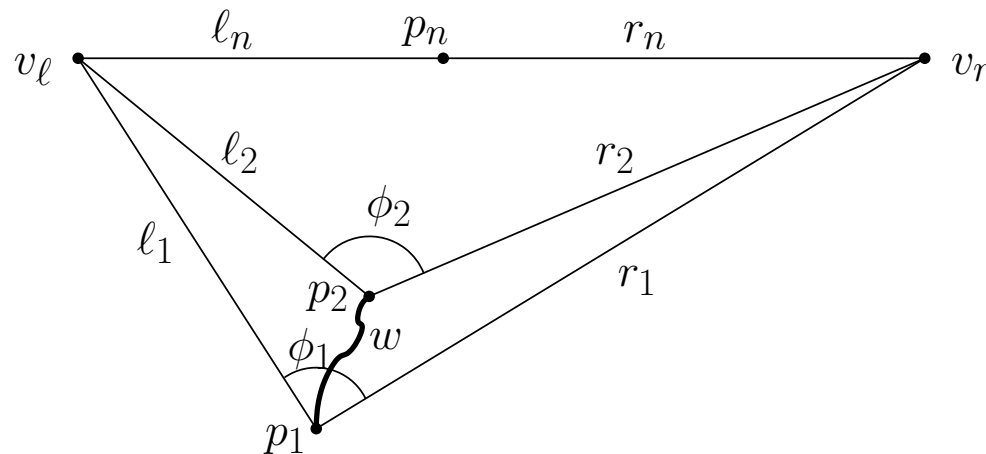
Rep.: Generalized LB for funnels

- **Lemma** LB for funnel of opening angle ϕ : $K_\phi := \sqrt{1 + \sin \phi}$. ■
- Strongly increasing: $0 \leq \phi \leq \pi/2$, Interval $[1, \sqrt{2}]$ ■
- Strongly decreasing: $\pi/2 \leq \phi \leq \pi$, Interval $[\sqrt{2}, 1]$ ■
- Subdivide: Strategy up to $\phi_0 = \pi/2$, Strategy from $\phi_0 = \pi/2$ ■



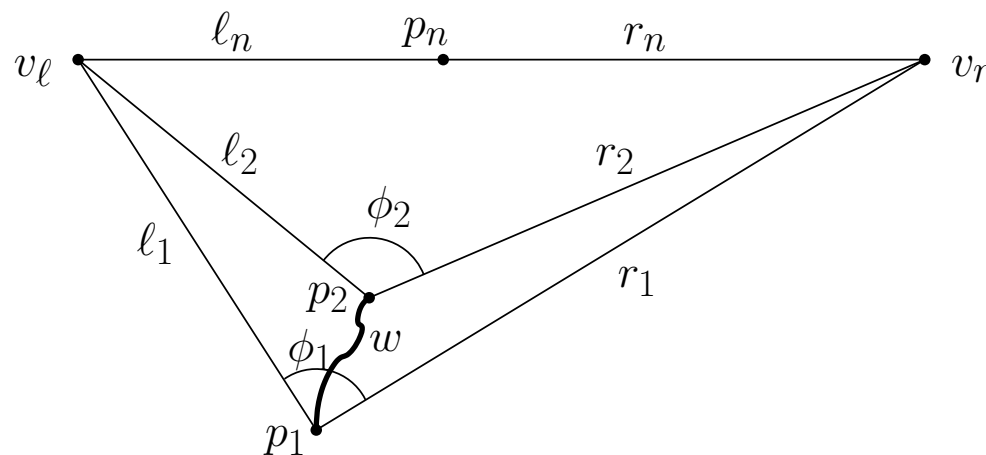
Rep.: Opt. strat. for angles $\pi \geq \varphi_0 \geq \pi/2!$

- Backward analysis: For $\varphi_n := \pi$ optimal strategy.■
- $K_\pi = 1$ and K_π -competitive opt. strategy with path l_n or r_n !■
- Assumption: Opt. strategy for some ϕ_2 with factor K_{ϕ_2} ex.■
- How to prolong for ϕ_1 with factor K_{ϕ_1} where $\frac{\pi}{2} \leq \phi_1 < \phi_2$?■
- We have $K_{\phi_1} > K_{\phi_2}$ ■



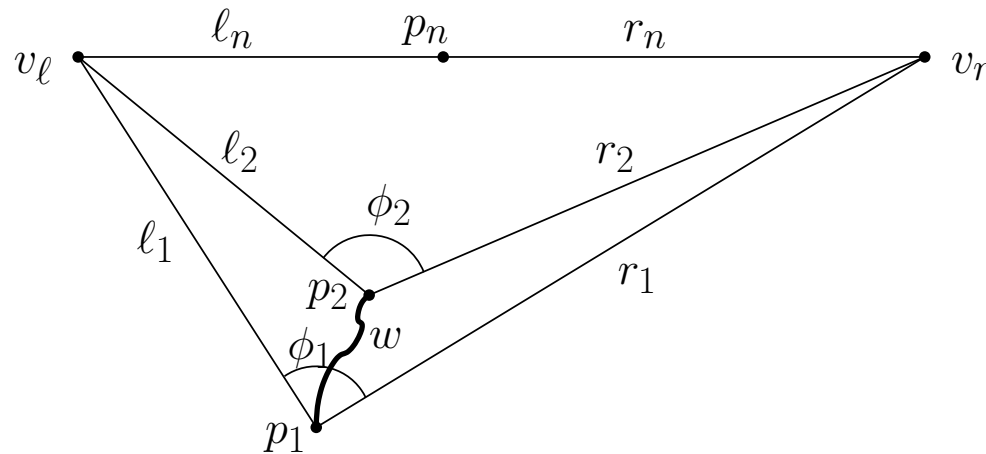
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- Situation: Opt. strategy for ϕ_2 with ratio K_{ϕ_2} ■
- How to get opt. strategy for K_{ϕ_1} ? ■
- Conditions for the path w ? Design! ■
- Goal behind v_l , path: $|w| + K_{\phi_2} \cdot \ell_2$, optimal: l_1 ■
- Goal behind v_r , path: $|w| + K_{\phi_2} \cdot r_2$, optimal: r_1 ■
- Means: $\frac{|w| + K_{\phi_2} \cdot \ell_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w| + K_{\phi_2} \cdot r_2}{r_1} \leq K_{\phi_1}$ ■



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- Guarantee: $\frac{|w| + K_{\phi_2} \cdot \ell_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w| + K_{\phi_2} \cdot r_2}{r_1} \leq K_{\phi_1}$ ■
- Combine, single condition for w ■
- $|w| \leq \min\{ K_{\phi_1} l_1 - K_{\phi_2} \ell_2, K_{\phi_1} r_1 - K_{\phi_2} r_2 \}$ ■
- Change of a vertex at p_2 ? ■ Remains guilty! ■



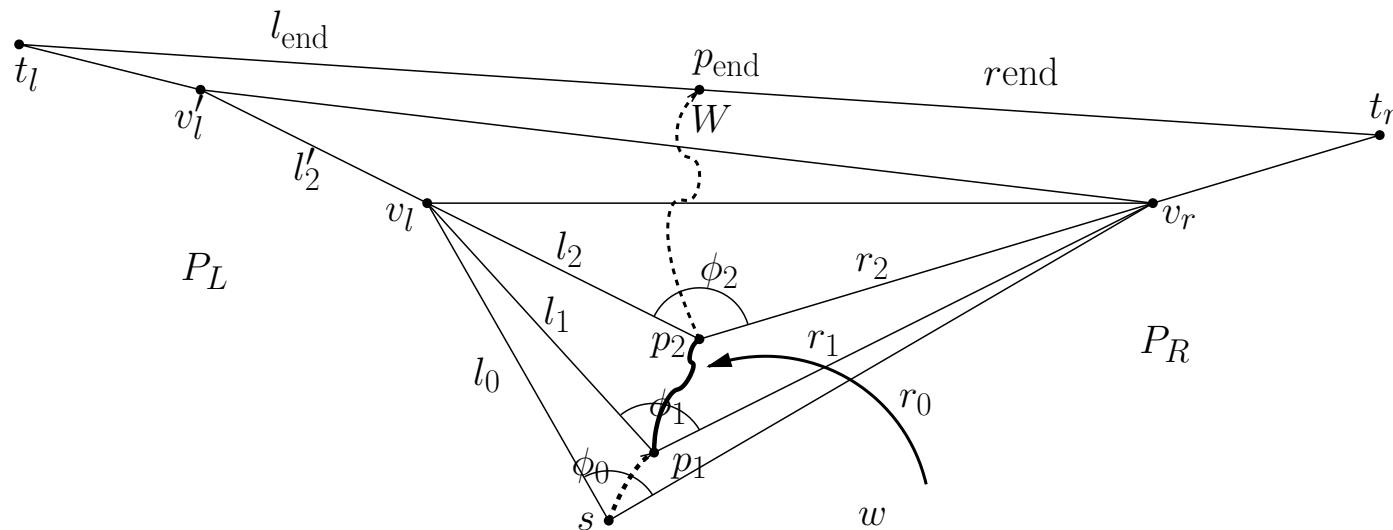
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- Change left hand: Condition

$$|w| \leq \min\{ K_{\phi_1} l_1 - K_{\phi_2} l_2, K_{\phi_1} r_1 - K_{\phi_2} r_2 \}$$

- There is opt. strategy for ϕ_2

- Show: $\frac{|w| + K_{\phi_2} \cdot (l_2 + l'_2)}{(l_1 + l'_1)} \leq K_{\phi_1}$

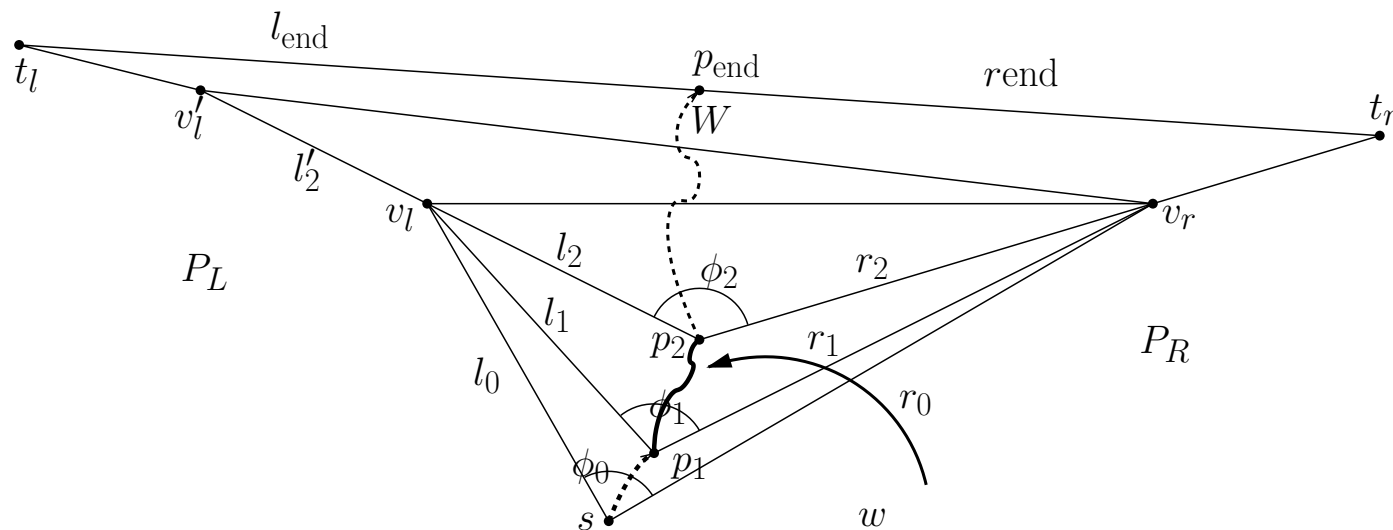


Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

$$|w| \leq K_{\phi_1} l_1 - K_{\phi_2} l_2$$

$$\blacksquare = K_{\phi_1} l_1 - K_{\phi_2} l_2 + K_{\phi_2} l'_2 - K_{\phi_2} l'_2$$

$$\blacksquare \leq K_{\phi_1} (l_1 + l'_2) - K_{\phi_2} (l_2 + l'_2) \blacksquare$$

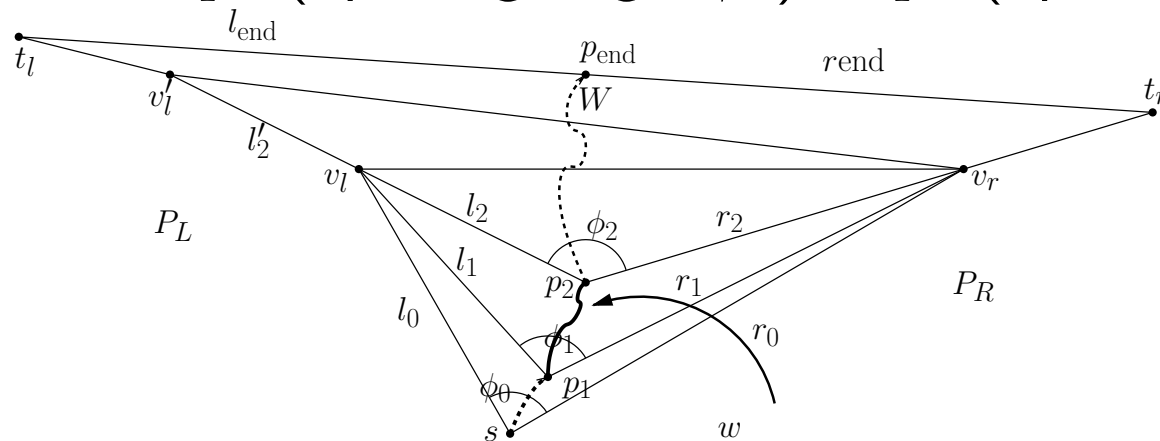


Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

Lemma Let S be a strategy for funnels with opening angles $\phi_2 \geq \frac{\pi}{2}$ and competitive ratio K_{ϕ_2} . We can extend this strategy to a strategy with ratio K_{ϕ_1} for funnels with opening angles ϕ_1 where $\phi_2 > \phi_1 \geq \frac{\pi}{2}$, if we guarantee

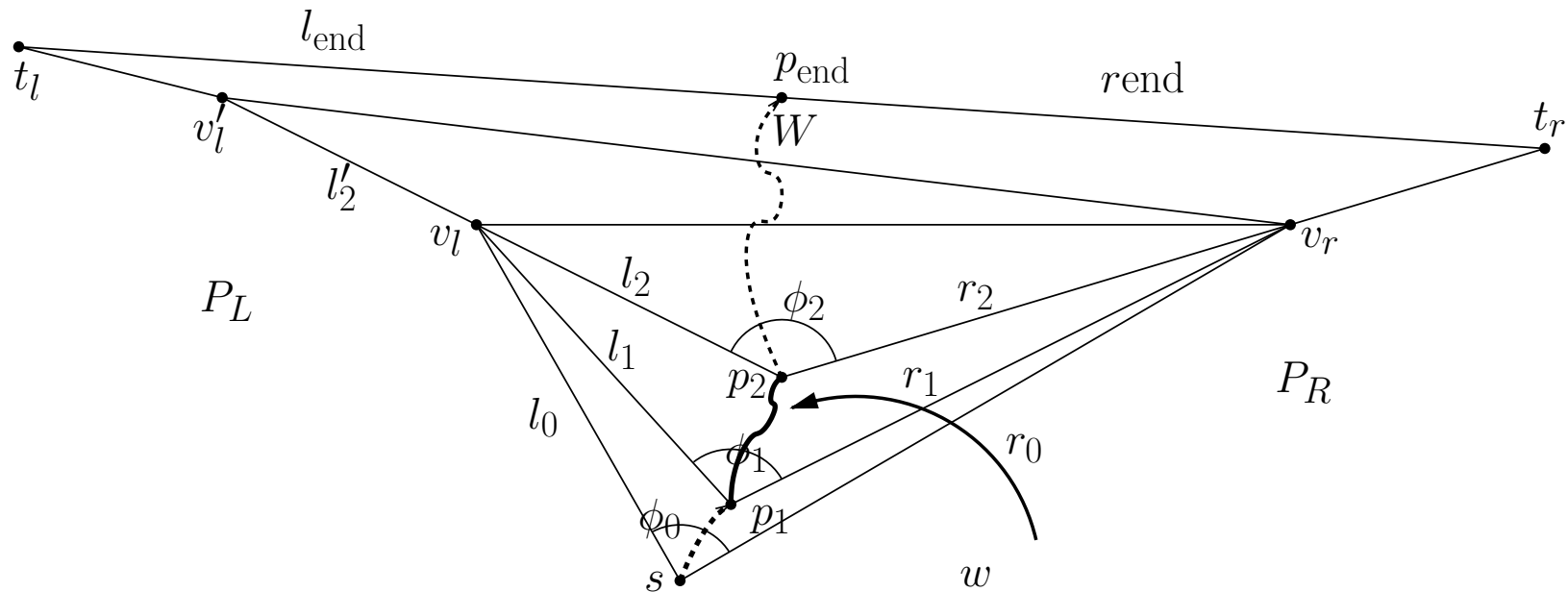
$$|w| \leq \min\{ K_{\phi_1} l_1 - K_{\phi_2} l_2, K_{\phi_1} r_1 - K_{\phi_2} r_2 \}$$

for the path w from p_1 (opening angle ϕ_1) to p_2 (opening angle ϕ_2). ■



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- If $|w| \leq \min\{K_{\phi_1}l_1 - K_{\phi_2}l_2, K_{\phi_1}r_1 - K_{\phi_2}r_2\}$ holds, then
- $|W| \leq \min\{K_{\phi_0} \cdot |P_L| - K_{\pi}l_{\text{End}}, K_{\phi_0} \cdot |P_R| - K_{\pi}r_{\text{End}}\}$. ■

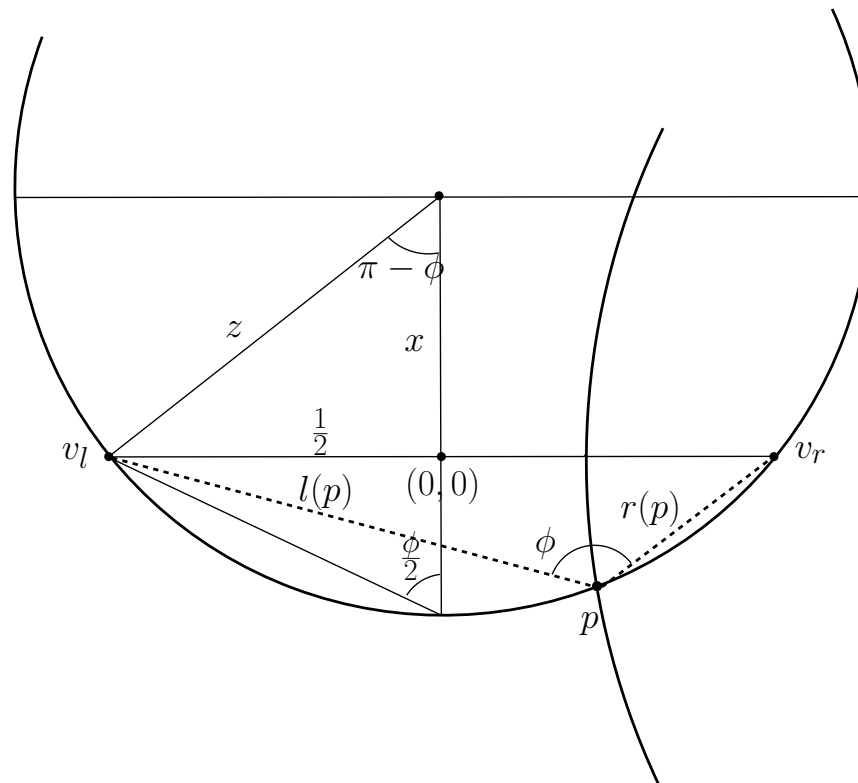


Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- $|w| \leq \min\{ K_{\phi_1} \ell_1 - K_{\phi_2} \ell_2, K_{\phi_1} r_1 - K_{\phi_2} r_2 \}$ ■
- How to fulfil this? ■
- Equality for both sides: $K_{\phi_2}(\ell_2 - r_2) = K_{\phi_1}(\ell_1 - r_1)$ ■
- Good choice for both sides! ■
- Defines a curve! ■
- We start with $A = K_{\phi_0}(\ell_0 - r_0)$ ■
- Parametrisation! ■

$$A = K_{\phi_0}(\ell_0 - r_0)$$

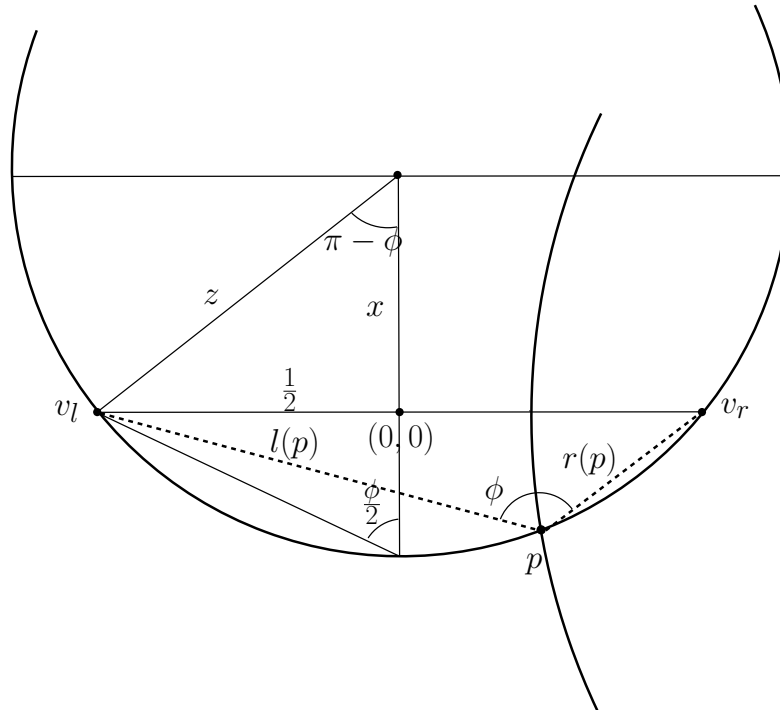
- Hyperbola: $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, $l - r = 2a$, $2c$, $a^2 + b^2 = c^2$
- Circle: $X^2 + (Y - x)^2 = z^2$, $r = z$, $(0, x)$



Intersection with circle and hyperbola

- Hyperbola: $\frac{X^2}{\left(\frac{A}{2K_\phi}\right)^2} - \frac{Y^2}{\left(\frac{1}{2}\right)^2 - \left(\frac{A}{2K_\phi}\right)^2} = 1$

- Circle: $X^2 + \left(Y + \frac{\cot \phi}{2}\right)^2 = \frac{1}{4 \sin^2 \phi}$

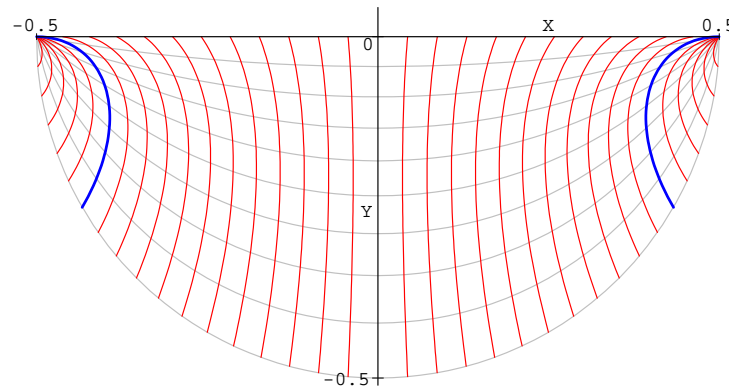


Opt. strat. for opening angle $\pi \geq \varphi_0 \geq \pi/2!$

Intersection: Verification by insertion! ■

$$X(\phi) = \frac{A}{2} \cdot \frac{\cot \frac{\phi}{2}}{1 + \sin \phi} \cdot \sqrt{\left(1 + \tan \frac{\phi}{2}\right)^2 - A^2}$$
$$Y(\phi) = \frac{1}{2} \cdot \cot \frac{\phi}{2} \cdot \left(\frac{A^2}{1 + \sin \phi} - 1\right)$$

where $A = K_{\phi_0}(\ell_0 - r_0)$ ■

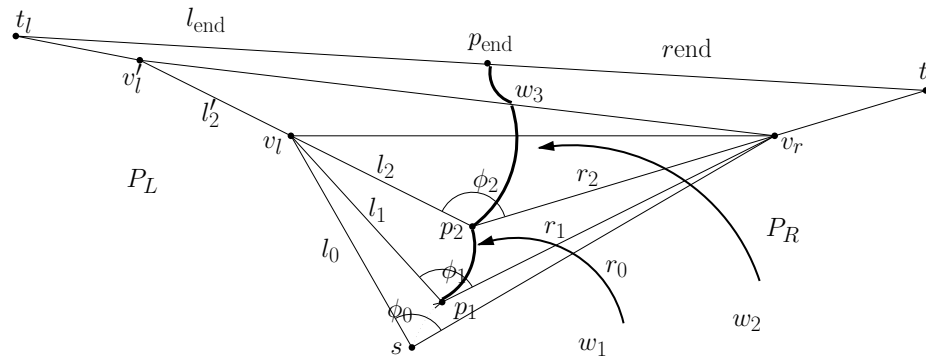


Opt. strat. for opening angle $\pi \geq \varphi_0 \geq \pi/2!$

$$X(\phi) = \frac{A}{2} \cdot \frac{\cot \frac{\phi}{2}}{1 + \sin \phi} \cdot \sqrt{\left(1 + \tan \frac{\phi}{2}\right)^2 - A^2}$$

$$Y(\phi) = \frac{1}{2} \cdot \cot \frac{\phi}{2} \cdot \left(\frac{A^2}{1 + \sin \phi} - 1\right)$$

Change of the boundary points. A also changes, new piece of curve!



Opt. strat. for opening angle $\pi \geq \varphi_0 \geq \pi/2!$

Theorem: The goal of a funnel with opening angle $\phi_0 > \frac{\pi}{2}$ can be found with ratio K_{ϕ_0} . ■

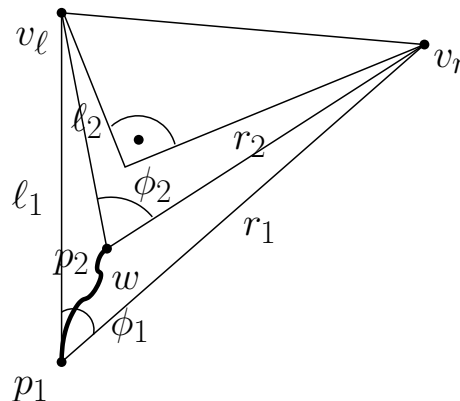
Proof: Show that the curves fulfil:

$$|w| \leq \min\{ K_{\phi_1} \ell_1 - K_{\phi_2} \ell_2, K_{\phi_1} r_1 - K_{\phi_2} r_2 \} \blacksquare$$

For any small piece w of the curve. ■ Analytically, lengthy proof!
■ Experimentally! ■

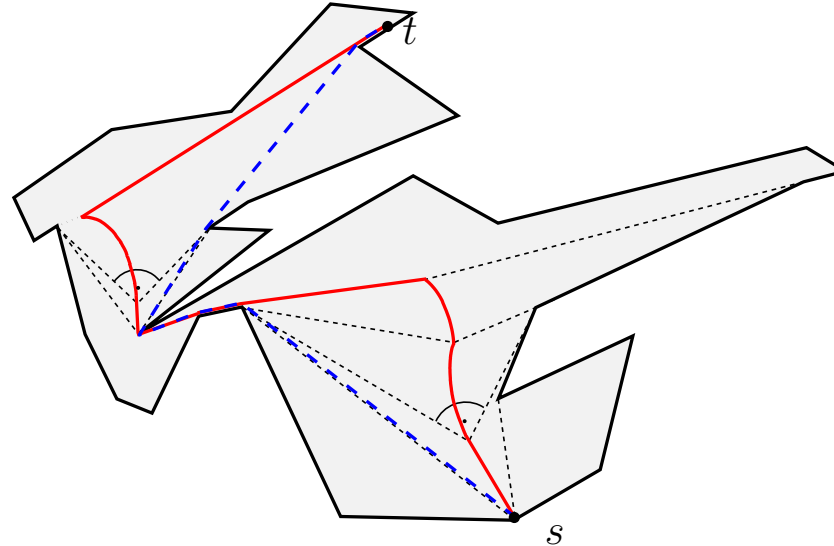
Opt. strat. opening angle $0 \leq \varphi_0 \leq \pi/2!$

- The same approach
- But independent from the angle
- Dominated by factor $K_{\pi/2} = \sqrt{2}$
- Require: $w \leq \min\{ \sqrt{2}(\ell_1 - \ell_2) , \sqrt{2}(r_1 - r_2) \}$.
- Equality: $\ell_1 - \ell_2 = r_1 - r_2$
- Current angular bisector: Hyperbola!



Opt. strat. opening angle $0 \leq \varphi_0 \leq \pi!$

Combine strategy 1 and strategy 2



Theorem: In an unknown street-polygon beginning from the source s we can find the target t with an optimal online strategy with competitive ratio $\sqrt{2}$.

Optimal strategy “Worst-Case-Aware”

As long as target t is not visible:

Compute current v_ℓ and v_r .

If only one exists: Move directly toward the other.

Otherwise. Repeat:

New reflex vertex v'_ℓ or v'_r is detected:

Use v'_ℓ or v'_r instead of v_ℓ or v_r .

Let ϕ be the angle between v_ℓ , the current position and v_r .

If $\phi \leq \frac{\pi}{2}$: Follow the current angular bisector!

If $\phi > \frac{\pi}{2}$: Follow the curve $(X(\phi), Y(\phi))$.

Until either v_ℓ or v_r is explored.

Move toward the non-explored vertex.

Move toward the goal.