Discrete and Computational Geometry Winter term 2016/2017
Exercise Sheet 04
University Bonn, Institute of Computer Science I

Deadline: Tuesday 15.11.2016, until 12:00 Uhr Discussion: 21.11. - 25.11.

- *Please give your solutions directly to the tutor or put them in the postbox at LBH next to E.01 until the deadline. Write your names well visible and readable on the first page. If your solutions consists of multiple pages, make sure they are well connected.*
- *It is possible to submit in groups of up to three people.*

Aufgabe 1: Application of Minkowskis Theorem (4 Points)

Consider the regular (5×5) lattice around the origin. Calculate the required expansion (radius r) of the *trees* at the lattice points so that any line $Y = aX$ hits at least one of the *trees*. Do the calculation in the following ways:

1. Calculate the radius r directly and precisely by considering the corresponding circles and lines.
   (W.l.o.g. only two cases have to be considered!)
2. Make use of the Minkowski Theorem and compute a non-trivial radius r that fulfills the requirement.

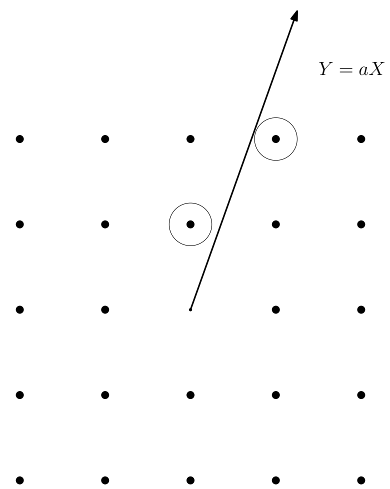

Abbildung 1: The regular (5×5) grid. The line passes the circles.

Aufgabe 2: Angles in Voronoi Diagrams (4 Points)

Let a, b and c be three distinct points in the plane that do not all lie on a line. Then their Voronoi-Diagram has a single Voronoi vertex v.

What can you say about the angles between the bisectors at v? Can they be any three angles α, β and γ as long as $\alpha + \beta + \gamma = 2\pi$?

If yes, prove it. If no, give a counterexample and give a more restrictive condition that always has to be fulfilled by the angles.

Aufgabe 3: Lower Bound (4 Points)

In the lecture it was shown that $\sum_{i=1}^{n} 1_i = O(n \log n)$. Show that $\sum_{i=1}^{n} 1_i = \Omega(n \log n)$ and hence $\sum_{i=1}^{n} 1_i = \Theta(n \log n)$.